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GUIDE TO ARITHMETIC,

OR, THE

ARITHMETICAL SELF-INSTRUCTER;

WHEREIN

THE PROPERTIES OF NUMBERS ARE CLEARLY POINTED OUT, AND THE THEORY AND PRACTICE OF THE SCIENCE DEDUCED FROM FIRST PRINCIPLES, AND DEMONSTRATED IN A FAMILIAR MANNER.

BY THOMAS WILLIAM FRYER,

AUTHOR (IN CONJUNCTION WITH DR. WILLCOLKES AND T. FRYER) OF THE SYSTEM OF ARITHMETICAL AND MENTAL CALCULATIONS.

WITH A KEY.

Price 2s. 6d.

1838

38.

103.

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AUTHOR'S ADDRESS TO THE PUBLIC.

THOUGH the Author is perfectly aware that works on Arithmetic are numerous, yet he does not think any apology necessary for the introduction of a new work on that highly important subject. As he has had a considerable degree of practice, both in lecturing and communicating Arithmetic to classes in schools, and to Mercantile and other individuals, who had obtained all the information that the ordinary system of tuition afforded, and also by the introduction of five editions (containing over 18,500 copies*) of the work alluded to in the title-page, he has generally found a very great deficiency, as regards a knowledge of the fundamental Rules; and being convinced that without a perfect knowledge of them it is idle to expect to attain any thing even bordering on proficiency, he is induced to endeavour to obviate the inconvenience by introducing a work, in his opinion, better suited for teaching the first principles of the science, and impressing them effectually on the pupil's mind, whether youth or adult, than any book that has yet come under his observation. Many of the ordinary Arithmetics are not sufficiently explicit; others do not give sufficient examples in Addition, Subtraction, Multiplication, and Division;

---

* The Sixth Edition of Dr. Willcolkes's and Messrs. Fryers' System of Arithmetical and Mental Calculations, was ready in January 1838, since which period several hundred copies have been disposed of; it contains *many hundred additional Calculations, not introduced into any of the five preceding Editions*, by which the size of the book is increased nearly 100 per cent., and will be found a work of the greatest importance, suiting both School and Counting-house.

others enter too abruptly into the science, and propose questions too difficult for the young tyro to manage; whereas they should proceed by the easiest possible stages; whilst others pride themselves, it would appear (to show their learning, it is supposed) by using a variety of technical terms, the meaning of which, the entire of children, and generality of adults, are totally ignorant of. To obviate these imperfections, and to furnish a system that will afford ease and information to the pupil, and a proportionate degree of satisfaction to the teacher, whilst it will enable those who have not time or inclination to employ an oral instructer, to instruct themselves, is the Author's object in sending the Work to press that now presents itself to your notice.

The pleasing reflection, that the name of Fryer, coupled with that of Dr. Willcolkes, is known to the Principals of *several hundred schools in this kingdom, in many of which personal instruction has been given*, besides their Work, "WILLCOLKES'S AND FRYERS' UNITED SYSTEM OF ARITHMETICAL AND MENTAL CALCULATIONS," being sanctioned by so many thousands of mercantile and professional gentlemen, gives no small degree of satisfaction to the Author, who confidently presents this, his well meant intentions, to an enlightened and liberal public, not fearing but patronage will be extended commensurate with his labours, and the utility of his publication.

London, May 1838.

ARITHMETICAL TABLES.

NUMERATION.

Units	1
Tens	1 2
Hundreds	1 2 3
Thousands	1 , 2 3 4
Tens of Thousands	1 2 , 3 4 5
C. of Thousands	1 2 3 , 4 5 6
Millions	1 ; 2 3 4 , 5 6 7
X. of Millions	1 2 ; 3 4 5 , 6 7 8
C. of Millions	1 2 3 ; 4 5 6 , 7 8 9

CURRENT MONEY.

4 Farthings make ... 1 Penny
12 Pence.................. 1 Shilling
20 Shillings, 1 Pound, or Sovereign

PENCE.

d.		*s.*	*d.*
20	are	1	8
24	...	2	0
30	...	2	6
36	...	3	0
40	..	3	4
48	...	4	0
50	...	4	2
60	...	5	0
70	...	5	10
72	...	6	0
80	...	6	8
84	...	7	0
90	...	7	6
96	..	8	0
100	...	8	4
108	...	9	0
120	...	10	0

SHILLINGS.

s.		*£.*	*s.*
20	are	1	0
30	...	1	10
40	...	2	0
50	...	2	10
60	...	3	0
70	...	3	10
80	...	4	0
90	...	4	10
100	...	5	0
110	...	5	10
120	...	6	0
130	...	6	10
140	...	7	0
150	...	7	10
160	...	8	0
170	...	8	10
180	...	9	0

MULTIPLICATION.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

Note, this table may be applied to division by reversing it: as the 2's in 4 are 2; the 2's in 6 are 3, &c.

LONG MEASURE.

3 barley-corns 1 inch
12 inches ...... 1 foot
3 feet ...... 1 yard
6 feet ...... 1 fathom
5½ yds. ...... 1 pole
40 poles ...... 1 furlong
8 fur. ...... 1 mile
3 miles ...... 1 league
69½ miles ...... 1 degree

LAND MEASURE.

9 feet make 1 yard
30¼ yds. ...... 1 pole
40 poles ...... 1 rood
4 roods ...... 1 acre

CLOTH MEASURE.

2¼ inch. make 1 nail
4 nails ...... 1 quar.
3 qrs. ...... 1 Fl. ell
4 qrs. ...... 1 yard
5 qrs. ...... 1 En. ell
6 qrs. ...... 1 Fr. ell

TIME.

60 sec. ... 1 minute
60 min. ... 1 hour
24 hours ... 1 day
7 days ... 1 week
4 weeks ... 1 month

DRY MEASURE.

2 gall. make 1 peck
4 pecks ... 1 bushel
4 bushels ... 1 sack
8 bushels ... 1 quarter
4 quarters ... 1 chaldron
10 quarters ... 1 last

𝔗𝔞𝔟𝔩𝔢𝔰 𝔬𝔣 𝔚𝔢𝔦𝔤𝔥𝔱𝔰 𝔞𝔫𝔡 𝔐𝔢𝔞𝔰𝔲𝔯𝔢𝔰.

PRACTICE.

OF A POUND, OR SOVEREIGN.

s.	*d.*		*£.*
10	0	are	1 half
6	8	...	1 third
5	0	...	1 fourth
4	0	...	1 fifth
3	4	...	1 sixth
2	6	...	1 eighth
2	0	...	1 tenth
1	8	...	1 twelfth
1	4	...	1 fifteenth
1	3	...	1 sixteenth
1	0	is	1 twentieth

OF A SHILLING.

d.		*s.*
6	are	1 half
4	....	1 third
3	....	1 fourth
2	....	1 sixth
1½	is	1 eighth
1	....	1 twelfth

OF A PENNY.

farth.		*d.*
2	are	1 half
1	is	1 fourth

OF A TON.

Cwt.		*T.*
10	are	1 half
5	...	1 fourth
4	...	1 fifth
2½	...	1 eighth
2	...	1 tenth
1	is	1 twentieth

OF A HUNDRED.

qr.	*lb.*		*Cwt.*
2	0	are	1 half
1	0	is	1 fourth
0	16	are	1 seventh
0	14	...	1 eighth

TROY.

24 gr. make 1 dwt.
20 dwt. .. 1 ounce
12 oz. ... 1 pound

AVOIRDUPOIS.

16 dr. make 1 oz.
16 oz. ...... 1 lb.
14 lb. ...... 1 stone
28 lb. ...... 1 quarter
4 qr. ...... 1 cwt.
20 cwt....... 1 ton

APOTHECARIES'.

20 gr. make 1 scruple
3 scr. .... 1 dram
8 dr. .... 1 ounce
12 oz. .... 1 pound

WOOL.

7 lb. make 1 clove
2 cloves .... 1 stone
2 stones .... 1 tod
6½ tods .... 1 wey
2 weys .... 1 sack
12 sacks .... 1 last

ALE AND BEER.

2 pints make 1 quart
4 quarts ... 1 gallon
9 gallons ... 1 firkin
2 firkins ... 1 kilder.
2 kilderkins 1 barrel
1½ barrel ... 1 hhd.
2 barrels ... 1 punch.
3 barrels ... 1 butt.

WINE.

2 pints make 1 quart
4 quarts ... 1 gallon
10 gallons ... 1 anker
42 gallons ... 1 tierce
63 gallons ... 1 hhd.
2 hhds. ... 1 pipe
2 pipes ... 1 tun

SOLID MEASURE.

1728 inches 1 solid foot
27 feet ... 1 yard

COAL MEASURE.

3 bushels ... 1 sack
36 bushels ... 1 chaldron

CUSTOMARY WEIGHT OF GOODS.

	lb.
A firkin of butter is	56
A firkin of soap	64
A barrel of pot ashes	200
A barrel of anchovies	30
A barrel of soap	256
A barrel of butter ...	224
A fother of lead, 19 cwt. 2 qrs. or	2184
A barrel of candles...	120
A stone of iron or shot	14
A gallon of train oil	7½
A fagot of steel	120
A stone of glass	5
A seam of glass 24 stone, or	120
A roll of parchment, 5 dozen skins	
A barrel of figs from nearly 96 to	360

MULTIPLICATION TABLE CONTINUED.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400

EXTENDED PENCE TABLE.

d.	£.	s.	d.	d.	£.	s.	d.	d.	£.	s.	d.	d.	£.	s.	d.	d.	£.	s.	d.
200 are	0	16	8	800 are	3	6	8	2,000 are	8	6	8	3,500 are	14	11	8	7,000 are	29	3	4
240 ...	1	0	0	900 ...	3	15	0	2,160 ...	9	0	0	3,600 ...	15	0	0	7,500 ...	31	5	0
300 ...	1	5	0	960 ...	4	0	0	2,400 ...	10	0	0	3,840 ...	16	0	0	8,000 ...	33	6	8
400 ...	1	13	4	1,000 ...	4	3	4	2,500 ...	10	8	4	4,000 ...	16	13	4	8,500 ...	35	8	4
480 ...	2	0	0	1,200 ...	5	0	0	2,640 ...	11	0	0	4,500 ...	18	15	0	9,000 ...	37	10	0
500 ...	2	1	8	1,440 ...	6	0	0	2,880 ...	12	0	0	5,000 ...	20	16	8	9,500 ...	39	11	8
600 ...	2	10	0	1,500 ...	6	5	0	3,000 ...	12	10	0	5,500 ...	22	18	4	10,000 ...	41	13	4
700 ...	2	18	4	1,680 ...	7	0	0	3,120 ...	13	0	0	6,000 ...	25	0	0	11,000 ...	45	16	8
720 ...	3	0	0	1,920 ...	8	0	0	3,360 ...	14	0	0	6,500 ...	27	1	8	12,000 ...	50	0	0

FRYER'S GUIDE TO ARITHMETIC,

OR, THE

ARITHMETICAL SELF-INSTRUCTER.

NOTATION AND NUMERATION.

ARITHMETIC is a science which explains the properties, and shows the use of numbers.

Unity, or a unit, is the number one; every other number is an assemblage of units.

For facilitating the management of numbers in Arithmetic, they are generally expressed by signs or characters; the method of expressing numbers by characters is termed NOTATION.

The method of discovering the value of numbers already expressed by characters, is called NUMERATION.

In modern Arithmetic all numbers are expressed by means of the following ten characters, called figures.

Cipher or Nought	One	Two	Three	Four	Five	Six	Seven	Eight	Nine
0	1	2	3	4	5	6	7	8	9

The values of these figures appear above at the first view; but, besides their simple values there expressed, they receive new values from the place they possess in numbers expressed by several figures.

The values of the places increase in a tenfold proportion infinite.

The first place is that next the right hand; the second, the next on the left of it, and so on.

Every figure standing alone expresses its own value; as 7 expresses the number seven, 9 nine, &c.

If a number be expressed by more figures than one, (as every number above nine is,) then the first figure signifies its own value, or units; the second ten times its own value, or as many tens as singly it expresses units, &c. Thus, in the following numbers:

17 Seventeen.
71 Seventy-one.
64 Sixty-four.

The first figures, 7, 1, 4, signify simply their own value, viz. 7 seven, 1 one, and 4 four; but the second figures, 1, 7, 6, receive a new value from their place, and stand for ten times their simple value, viz. 1 signifies ten, 7 seventy (seven tens,) and 6 sixty (or six tens)

In like manner, a figure in the third place signifies ten times ten times, or a hundred times as much as it would in the first; as in the numbers,

317 Three hundred and seventeen.
571 Five hundred and seventy-one.
764 Seven hundred and sixty-four.

And so continually every figure to the left hand has ten times the value it would have in the next place to the right, and signifies ten times its own value so often repeated as its place is distant from the first or units' place.

Whence it follows, and is easy to be conceived, that every ten of the value of a lower place is equal to one of the next higher.

But as the frequent repetition of tens would occasion obscurity and confusion, different terms are used to express the local value of figures; as by the following

TABLE.

	Place.	Name.
First Period.	First	Units
	Second	Tens X
	Third	Hundreds C
	Fourth	Thousands......M
	Fifth	X of Thousands
	Sixth	C of Thousands
Second Period.	Seventh	Millions
	Eighth	X of Millions
	Ninth	Hundreds of Millions
	Tenth	Thousands of Millions
	Eleventh	X of Thousands of Millions
	Twelfth	C of Thousands of Millions
Third Period.	Thirteenth	Millions of Millions, or Billions
	Fourteenth	X of Billions
	Fifteenth	C of Billions
	Sixteenth	Thousands of Billions
	Seventeenth	X of Thousands of Billions
	Eighteenth	C of Thousands of Billions
	Nineteenth	Millions of Billions, or Trillions

Under every sixth place we draw a line of separation, because the next place (according to the preceding progression of the places) would be thousands of thousands; but again, to prevent the obscurity that the frequent repetition of the same word would occasion, we use the term millions, to stand for a thousand thousands, and then proceed to name the next five places successively, by prefixing to the word millions the same terms, in order, as in the five places after units; and so on in a continual rotation: every round of six places is named a Period, and the table may be yet continued further at pleasure, by learning names for the lowest place of each succeeding period, as in the following

TABLE.

Of the lowest place of the	Period, the name is
First	Units
Second	Millions
Third	Billions
Fourth	Trillions
Fifth	Quadrillions
Sixth	Quintillions
Seventh	Sextillions
Eighth	Septillions
Ninth	Octillions, &c.

Likewise each Period may be subdivided into two equal parts, which may be called members, consisting of three places each.

OF THE CIPHER.

A cipher to the left hand of any number does not alter its value, since every significant figure continues to possess the same place; as,

45
045
0045, &c.

But if a cipher be put to the right hand of any number, it increases its value ten times, by removing every figure one place higher: two ciphers placed in like manner cause the significant figures to express a number one hundred times as much as without the ciphers, since every figure is removed two places higher by such positions of the ciphers; as,

45	Forty-five.
450	Four hundred and fifty.
4500	Four thousand five hundred.

NUMERATION TABLE.

C of Millions.	X of Millions.	Millions.	C of Thousands.	X of Thousands.	Thousands.	Hundreds.	Tens.	Units.
9	8	7	6	5	4	3	2	1

Observe that the 0 by itself signifies nothing; being put on the *left* hand of any number it does not alter its value; but being put to the *right* hand, it makes the number *ten* times what it was before.

EXAMPLES FOR PRACTICE.

Write down in figures

Five............5
Nine............
Fourteen ...14
Thirty-five...
Eighty-eight.
One hundred and sixty-five.........165
Two hundred and seventy-one.
Seven hundred and eighty-three.
Nine hundred and ten.
One thousand two hundred and twenty-four... 1224
Two thousand seven hundred and forty-eight.
Eight thousand two hundred and fifty-three.
Nine thousand one hundred and fourteen.
Nine thousand nine hundred and ninety-nine.
Ten thousand seven hundred and twenty-four...10724
Twelve thousand two hundred and ninety-five.
Fifteen thousand four hundred and five.
Twenty-seven thousand nine hundred and forty-six.
Eighty-six thousand five hundred and fourteen.
Ninety-five thousand one hundred and eighty-seven.
One hundred and twenty-seven thousand three hundred and ninety-four................................127394
Three hundred and twenty-nine thousand eight hundred and forty-seven.
Six hundred and two thousand four hundred and nine.
Five hundred and forty thousand eight hundred and five.
Six hundred and twenty-seven thousand and twenty-eight.
Four millions one hundred and twenty-seven thousand three hundred and twenty-six 4127326
Six millions four hundred and forty-seven thousand nine hundred and one.

Nine millions fifty thousand and eighty ...9050080

Sixty millions one hundred and forty-seven thousand six hundred and two.

Nine millions.

Seven thousand.

Six hundred.

Note. The teacher can propose any other numbers for the pupil to practise, as he or she finds necessary.

Numeration is also the right reading of, or the conversion of figures into words.

Rule. Consider the value each figure receives from its place; thus, read four in the first place on the right, as 4 in the second, as 40 in the third, as 400, &c. &c.; but that the pupil will learn as he proceeds in the Rules that follow.

Obs. The introductory Rules of Numeration and Notation are seldom well understood by young beginners, and it is frequently a considerable time before they apprehend rightly the respective values of every individual place. It might therefore be sufficient, for the present, if the junior pupils were taught to read or express with facility, three or four places of figures, units, tens, hundreds, and thousands; as, 17, 71, 132, 144, 1826, &c., which would enable them to go through the four first Rules in Integers, at the end of which, the Rules of Numeration and Notation may be studied with greater advantage by the pupil.

SIMPLE ADDITION.

The object of Addition is to find the number which is equivalent to two or more given numbers taken together.

The number which is equivalent to two or more numbers taken together, is called their sum.

When the given numbers are all of the same denomination, as all yards, or all gallons, the operation is termed Simple Addition.

When the numbers to be added express quantities of the same kind, but of different denominations, the operation is termed Compound Addition.

TO ADD QUANTITIES OF THE SAME DENOMINATION.

Rule 1. Place the numbers so that units may stand under units, tens under tens, &c. 2. Find the sum of the column of units, set down the last figure of it below that column, and carry to the next the number expressed by the remaining figure, or figures, if there be any. 3. Proceed as before with the remaining columns, and at the last column set down the entire sum.

Thus, to add together 9468, 2956, and 79, let them be set as in the margin, and proceed thus:—9 and 6 are 15, and 8 are 23; set down 3, and carry 2 to the column of tens. Then 2 and 7 are 9, and 5 are 14, and 6 are 20; set down a cipher, and carry 2. 2 and 9 are 11, and 4 are 15; set down 5, and carry 1. 1 and 2 are 3, and 9 are 12; set down 12, and the sum, or answer required, is twelve thousand five hundred and three.*

```
  (1)
 9468
 2956
   79
 ─────
12503 Sum.
```

REASON OF THE RULE.

The rule for performing addition depends on the nature of Notation, and on the obvious principle, that *the whole is equal to the sum of all its parts.* By placing units under units, tens under tens, &c. we are enabled the more easily to add together the figures of the same local value; and *one* is carried for every *ten*, because, by the nature of Notation, *ten* in any column is equivalent only to *one* in the column immediately to the left of it. We commence with the units merely for the convenience of carrying to the next columns. Thus, in the preceding example, the sum of the column of units is 23; and therefore, after setting down 3, we have 20 remaining. But, by the nature of Notation, 2 in the next column is equal to 20 in this; and therefore we carry only 2.

---

* The sign +, commonly called *plus*, is employed in Arithmetic and other parts of mathematics, to signify that the quantities between which it is placed are to be added together; and the sign, =, called *the sign of equality*, is used to denote, that the quantities between which it stands, are equal to one another. Thus, the expression, $12+9=21$, means, that 12 and 9 added together, are equal to 21, or that the sum of 12 and 9 is 21.

Some teachers may, perhaps, consider it proper to make pupils commit the following table to memory:—

ADDITION TABLE.

2 and	2 and	3 and	4 and	5 and	6 and	7 and
2 are 4	9 are 11	7 are 10	6 are 10	6 are 11	7 are 13	9 are 16
3 ... 5		8 ... 11	7 ... 11	7 ... 12	8 ... 14	
4 ... 6	3 and	9 ... 12	8 ... 12	8 ... 13	9 ... 15	8 and
5 ... 7	3 are 6		9 ... 13	9 ... 14		8 are 16
6 ... 8	4 ... 7	4 and			7 and	9 ... 17
7 ... 9	5 ... 8	4 are 8	5 and	6 and	7 are 14	9 and
8 ... 10	6 ... 9	5 ... 9	5 are 10	6 are 12	8 ... 15	9 are 18

To enable the learner to acquire accuracy and despatch in addition, it is proper to train him to add in the following manner, till he can do it with facility:—Since 6 and 6 are 12, 26 and 6 are 32: (here it should be pointed out to him, that 12 and 32 end in the same figure:) since 9 and 7 are 16, 39 and 7 are 46: since 8 and 6 are 14, 88 and 6 are 94: since 6 and 9 are 15, 16 and 9 are 25, &c.

METHODS OF PROOF.

1. Add the several columns, according to the rule, commencing at the top and proceeding downward, and if the result be the same as was obtained by adding them upward, it may be presumed that the work is right.

2. Separate the given numbers into two or more divisions.—Find the sums of these divisions, severally, and add these partial sums together. If the last result be equal to that found by the common method, the work is right.

This will appear obvious from the following example:

37928
93640
———
23574
75849
———
Entire sum.............. 230991
———
Sum of the 1st division 131568
Sum of the 2d division 99423
———
Entire sum.............. 230991 *Proof.*

This method may also be employed with advantage in finding the sums of large columns, instead of adding the whole at a single operation, The first method is very convenient and useful, when the columns are not very large.

3. Commencing at the left hand, add the several columns without *carrying*, and set down the full sum of each column with the units in their proper place, and the tens below the figure immediately to the left. Add together the two lines thus resulting, and if the last result agree with that obtained by the common method, it may be concluded that both are right.

Thus, in the annexed example, the sum of the left hand column is 25, which is set down in full: the sum in the next column is 30; the cipher is set in its proper place, and 3 under the 5; and so with the rest. The sum of the two lines thus obtained, is equal to the sum found by the ordinary method.

$$\begin{array}{rl} 5946 & \\ 9738 & \\ 2697 & \\ 9868 & \\ \hline 28249 & \textit{Sum.} \\ \hline 25029 & \\ 322 & \\ \hline 28249 & \textit{Proof.} \end{array}$$

This method of addition might be used instead of the common method; and as it requires nothing to be carried, it might be employed with advantage when the calculator is liable to interruptions.

FURTHER EXAMPLES.

ADD together 3415 and 6723. Thus:

$$\begin{array}{r} (2) \\ 3415 \\ 6723 \\ \hline \\ \hline \end{array}$$

Having taken care to place units under units, tens under tens, &c. and drawn a line, you commence at the right hand side, and say, 5 and 3 are 8, put it down; not amounting to ten, you have not to carry; then 2 and 1 are 3, put that down also, *under the figures that produce it;* then 7 and 4 are 11, you put down only the first figure on the *right*, and carry the other, which is 1, to 6 and 3, making 10; that being the last, you find the addition of the two numbers to be 10138, which is correct.

In the same way add the following :

(3)	(4)	(5)	(6)	(7)
4563	4827	97635	468759	4763281
8276	5638	4930	491234	5072987

If the learner cannot find the results of the foregoing easily, the teacher must propose some more, according to judgement, but if the above are considered sufficient, let the following be practised.

(8)
48763
29876
48358

The pupil will say, 8 and 6 are 14, and 3 are 17 ; put down the 7, and carry 1 to the next line, saying, 1 and 5 are 6, and 7 are 13, and 6 are 19 ; put down the 9 as before, and go to the next, saying, 1 and 3 are 4, and 8 are 12, and 7 are 19 ; just act as before ; having 1 again to carry, say, 1 and 8 are 9, and 9 are 18, and 8 are 26 ; put down the 6, and carry *two* this time to 4 are 6, and 2 are 8, and 4 are 12 ; the result, if you are correct, will be 126997.

In the same way add the following :

(9)
476593
385678
·6·2·1

Here, to assist the pupil, I have put the correct figures where they occur, leaving him to put the remainder.

I shall adopt the same method as I proceed, as it will enable the child at once to find out when he or she is not right.

	(10)	(11)	(12)	(13)
Add	49763	4327682	2548	476356
and	48275	5683297	2696	889643
and	50076	4831190	334	482976
	1·8··4	1··4··6·	8988	100680
			·4··6	··4··5·

Required the sum of 31614 + 4002 + 96183 + 816 + 60859 + 1678.

Remember that + is the sign of Addition.	(14)	
	31614	
	4002	
	96183	
	816	
	60859	
	1678	
	195152	*Sum.*

When practice has familiarised the operation of adding numbers together, it is better, in finding the amount of each row, to omit naming the several figures. Thus, instead of saying, 8 and 9 are 17, 17 and 6 are 23, and so on; an expert Arithmetician will only express the amounts every time as his eye passes over the figures. Thus:

8, 17, 23, 26, 28, 32. Set down 2, and carry 3.
3 and 7 are 10, 15, 16, 24, 25. Set down 5, and carry 2.
2 and 6 are 8, 16, 24, 25, 31. Set down 1, and carry 3.
3 to 1 are 4, 10, 14, 15. Set down 5, and carry 1.
1 to 6 are 7, 16, 19; which, placed under its own column, gives 195152, the sum required.

(15)	(16)
45687932	86957380
86957380	45687932
29765493	40006570
40006570	29765493
·0··1··7·	2··4··3··

In these two questions, having added up the four lines, and placed the product at the bottom, cut off the top line, and place the result of the other three under it; then add the result of the top line (cut off) to it, and if it agrees with the first line of your addition you may take it for granted the sum is right. Take the following for a more clear explanation.

(17) 45673
82945
56973
185591
139918
185591

I find the result of the sum to be 185591. I then cut off the top line, 45673, and add the two other lines, and place the product a little lower down than 185591; I find it to be 139918. I then add it and the top line together, and find it to be 185591, which agrees with the fir addition, and am satisfied I am correct. The same mode of proof can be applied to all addition sums, whether simple or compound, and will answer the end of working *three* sums instead of *one*. Adopt the same plan with all the following.

(18)	(19)	(20)	(21)
45638274	497638549	763295814	54963274
82960172	872365842	423615978	4932735
27106928	497632235	876594123	4827000
45873262	276056382	341298765	4923650
86559988	950068329	291038476	19

(22)	(23)	(24)	(25)
34618309	73487623	82998763	67236875
41872571	43194784	23765436	35976329
74139684	24687829	87349235	64136897
33657423	72358967	98984687	33297978
41936170	67243884	85239496	12315634
58674392	45892346	67148592	67854567

(26)	(27)	(28)	(29)
54215734	46877392	38478527	24346749
67534216	43518613	65924368	50419364
46813983	69732841	74359735	61942852
74641357	48976139	84563789	72634279
31257639	21643675	62325278	83647746
85632145	57320596	57396845	77360725

(30)	(31)	(32)	(33)
94135276	10256798	71634159	46754535
65916741	83124079	43217698	54548374
93061577	71032674	56716781	35474168
21537468	13180192	67634167	84796567
64195258	32594876	12467136	58467346
53871567	42197120	78467876	49375388

(34)	(35)	(36)	(37)
56432735	82621921	41965748	94517968
45457454	40212318	19286868	61785629
97545833	34227906	44758136	79857437
46818497	67572417	95471328	81362794
64967336	21123576	98135279	74135268
82546479	88839463	31426895	49615985
35731842	42372885	67531426	53728096

THE USE OF ADDITION.*

(38) A merchant on settling his accounts finds he owes A £60.; B £180.; C £247.; and D £385.; how much money will he require to pay the entire?

(39) What is the total weight of these six loads of iron, number 1 weighing 495 pounds; number 2, 623 pounds; number 3, 1468 pounds; number 4, 1283 pounds; number 5, 3296 pounds; and number 6, 1095 pounds.

(40) In an army consisting of 4560 foot soldiers, 4823 cavalry, 5675 light infantry, 1327 gunners, 1500 pioneers, and 200 miners, what number of men in the whole?

(41) A draper received from London the following bales of goods: of linen 4696 yards; of muslin 5693 yards; of flannel 1007 yards; and of calico 14763 yards; how many yards in the entire?

(42) A gentleman in his will left his property in the following way; to his wife £19347.; to his eldest son £9764.; to his second son £6000.; and to *each* of his *three* daughters £765.; I demand what his property amounted to.

(43) Suppose a man has a family of 8 children; his first has lived 7300 days; his second 6975 days; his third 6183 days; his fourth 5684 days; his fifth 5381 days; his sixth 4987 days; his seventh 4000 days; and his eighth 3412 days; how many days will all make?

Note. As there will be much practice in Addition as we proceed, the foregoing information, it is believed, will be found sufficient for the present.

SIMPLE SUBTRACTION.

THE object of SUBTRACTION is to find the difference between two numbers.

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* The questions under the above head will be useful to give the pupil an idea of business, as also to afford questions to be inserted in the ciphering book; the foregoing may also be inserted, as it will give practice in making figures, besides other advantages.

The number found in Subtraction is called the REMAINDER, the DIFFERENCE, or the EXCESS.*

When the given numbers are of the *same denomination*, the process is termed SIMPLE SUBTRACTION.

When the given numbers express quantities of the *same kind*, but of *different denominations*, the process is termed COMPOUND SUBTRACTION.

RULE FOR SIMPLE SUBTRACTION.

1. Place the less number below the greater, with units under units, tens under tens, &c. as in Addition. 2. Beginning with the units, take, if possible, each figure in the lower line, from the figure above it, and set down the remainder. 3. But if any figure in the lower line be greater than the figure above it, add ten to the upper; then subtract as before, and carry one to the next figure in the lower line.

The sign —, generally called *minus*, when set between two numbers, denotes that the latter is to be taken from the former. Thus: $16 - 9 = 7$, denotes, that if 9 be taken from 16 the remainder is 7.

METHODS OF PROOF.

1. Add the remainder and the less of the given numbers together: if the sum be equal to the greater, the work is correct.

2. Subtract the number found from the greater of the given numbers; if the remainder be equal to the less, the work is correct.

EXAMPLES.

From 7854 take 4513.

Set the numbers as in the margin, and proceed thus: 3 from 4, and 1 remains; 1 from 5, and 4 remain; 5 from 8, and 3 remain; 4 from 7, and 3 remain: the remainder therefore is 3341.

(1)
7854
4513
Remainder 3341
Proof 7854

To prove the work, to the less of the given numbers add the remainder, and the sum will be 7854, the greater; or, as in the second method, subtract the remainder from the greater number, and the result will be 4513, the less.

(2)
7854
4513
Remainder 3341
Proof 4513

---

* The number to be subtracted is sometimes called the *sub-end*, and that from which it is to be taken, the *minuend*.

Required the difference of 3712 and 1831.

In this example proceed thus:—1 from 2, and 1 remains; 3 from 11, and 8 remain: carry 1 to 8, and then 9 from 17, and 8 remain; carry 1, and then 2 from 3, and 1 remains.—The difference, therefore, is 1881, and the operation would be proved in the same manner as before.

(3)
From 3712
take 1831
Remainder 1881

REASON OF THE RULE.

The rule for Subtraction depends on the principle, that the differences of the several parts of two numbers are, when taken together, equal to the difference of the numbers themselves. The reason of placing units under units, tens under tens, &c. is, that figures may be subtracted from others of the same local value with more facility. By carrying one to the lower figure, we increase the lower line as much as we increased the upper, and thus the difference will be the same as if neither had been increased. Thus, in the second of the above examples, when in the tens' place we subtract 3 from 11, we thus add 10 to the 1 in the upper line; then the lower line is increased by the same quantity by adding 1 to the 8; because, by the nature of notation, 1 in the third column is equivalent to 10 in the second. Thus, therefore, both the given numbers are equally increased, and consequently the difference must be the same as if they had received no increase.

As a farther illustration of Subtraction, let it be required to find the difference of 83 and 57. Here, as 7 cannot be taken from 3, we may consider 83 as equal to 70 and 13; and subtracting 7 from 13, and 5 from 7, we find the difference to be 26. In this simple and natural method, the values of the given numbers undergo no change; and, with only one exception, it might be employed with as much facility as the common method, the next figure in the upper line being always diminished by a unit, when one would be carried to the figure below it, in the common method. The exception is the case in which the next figure in the upper line is a cipher:

(4)
83
57
26

in this case the common method is considerably preferable: and, as in practice, that method is in no case inferior, it is universally preferred.

EXERCISES.

(5)
Subtract 4567 from 9879
4567

You place the 4567 under the 9879, and, having drawn a line, you commence subtracting *units* from *units*, *tens* from *tens*, &c. &c., saying, 7 from 9, and 2 remain; put it down; 6 from 7, and 1 remains; 5 from 8, and 3 remain; and 4 from 9, and 5 remain; making the difference 5312. In that case you had not occasion to *borrow*, but in the following you will find it necessary to do so; you will have to *pay* again, so as to balance.

(6)
From 8547
take 7628

Here you are to say, 8 from 7, I cannot; but 8 from 17, and 9; put down 9: you then have to pay, by adding 1 to the 2, and say, 3 from 4, and 1; put it down: you have *not* to add any thing to 6, as you have not borrowed, as you consider the 4 as *four*, not fourteen; and 6 from 5, I cannot, therefore borrow: say 6 from 15, and 9; you will then have to pay one to the 7, making it 8, and say, 8 from 8, and nothing; put a 0 or a dot; thus, you will find the difference to be ·919.

Note. Simple Subtraction is a very easy Rule; the only thing requiring much attention is to remember, how you are to *borrow* and *pay*.

(7)
From 4976
take 0143

(8)
From 8976
take 4839

(9)
From 4326
take 28

Here you say, 8 from 6, you cannot; but having borrowed 10, you say, 8 from 16, and 8; place it under the *units*, and having borrowed, pay one back to the 2, making it 3; 3 from 2 you cannot, but 3 from 12, and 9; put that in the place for *tens*; having borrowed, you then say, 1 from 3, and 2; though there is not any figure under the 3, you must imagine one, and the 4 you place down, not having any thing to take from it either borrowed or otherwise.

*QUESTIONS TO PRACTISE SIMPLE SUBTRACTION.

(10) Le Place, the celebrated French Mathematician and Philosopher, was born in 1749; required what his age would have been in 1838 (the present year.)

(11) Mont Blanc is 15680 feet high, and Chimborazo, in America, is 21427 feet in height; how much higher is the latter than the former?

	(12)	(13)	(14)
From	633769345927	461420664315	751391748626
take	568520429365	337647584769	471234576146
	(15)	(16)	(17)
From	764186427649	512714137043	800147012376
take	526684789676	199741527649	178648607649
	(18)	(19)	(20)
From	239567496689	987654321134	743118653933
take	187698507190	958826947629	669206683569
	(21)	(22)	(23)
From	764111456672	310642264214	164176412347
take	618147157669	160777417656	67147671479
	(24)	(25)	(26)
From	369450004231	549326153340	562010325064
take	157689546782	84932589362	364256234546
	(27)	(28)	(29)
From	841012764276	899994100000	476427641476
take	61476427476	105605099999	17747647684
	(30)	(31)	(32)
From	613941630716	334561364564	718941363124
take	510198765437	178769543671	167846716781

---

* Whilst the pupil is learning Addition and Subtraction, the Multiplication Table, pages 5 and 6, should be got off by rote. I give two forms of Multiplication Table, that, that which may be considered the easiest to understand, may be selected.

	(33)	(34)	(35)
From	900662211552	100364219675	402680086786
take	489763462374	98765469787	160987658939

	(36)	(37)	(38)
From	452633266173	328795027503	397292602541
take	73005365592	52607204426	62730496498

SIMPLE MULTIPLICATION.

MULTIPLICATION is a compendious method of performing Addition, and teaches to find the amount of any given number, which requires to be repeated a certain number of times.

The number to be repeated is called the MULTIPLICAND; the number which shows how often the multiplicand is to be repeated, is called the MULTIPLIER; and the number found is called the PRODUCT.

When the multiplicand expresses a quantity of only *one denomination*, the process is termed SIMPLE MULTIPLICATION.

When the multiplicand expresses a quantity of the same kind, but of *more denominations than one*, the process is termed COMPOUND MULTIPLICATION.

MULTIPLICATION TABLE.*

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

---

* Though the part of the Multiplication Table given above is quite enough for the pupil to commit to memory at first, yet, after he has made some proficiency in Arithmetic, it may be found advantageous to get him to commit to memory the extended Multiplication Table, page 6.

It will perhaps enable the pupil to commit the preceding Table to memory with more ease, if he be caused to construct it by Addition.—Thus, to find the products by 7, let him set in one column the figure 1 seven times, in another the figure 2 seven times, in another the figure 3 as often, &c. Then, the sums of these columns will be the products by 7. He will thus be taught the construction and nature of the Table, and will perhaps find it easier and more interesting to commit the products which he has formed himself, than those which he finds without explanation in the Table. It will also assist the learner, if his attention be turned to the relations subsisting between some of the successive products in the Table. Thus, he will see that the products by 10 are formed simply by the addition of a cipher; that the first nine products by 11 are formed by repeating the figure; that the products by 5 terminate in 5 and 0, alternately; that in the products by 9, the first figure generally increases, and the second decreases, by a unit, &c.

RULE FOR SIMPLE MULTIPLICATION.

1. Place the multiplier below the multiplicand, with units under units, tens under tens, &c. 2. If the multiplier do not exceed 12, multiply, by means of the Multiplication Table, each figure of the multiplicand by it, beginning with the units, and setting down and carrying as in Addition. The result will be the product required. 3. But if the multiplier be greater than 12, find the products of the multiplicand by the several figures of the multiplier, successively; setting the right hand figure of each product *under that figure of the multiplier which produces it.* The sum of all these products will be the total product required.

If either the multiplicand, or the multiplier, or both, *end in ciphers,* the significant figures may be arranged and multiplied according to the rule, and as many ciphers annexed to the product, as are found at the end of *both factors.*—Ciphers in any other part of the multiplier, are to be neglected.

The sign ×, called *the sign of Multiplication,* is frequently used to denote the multiplication of the numbers between which it is placed. Thus, $20 \times 12 = 240$, denotes, that the product of 20 and 12 is 240.

EXAMPLES.

Multiply 324 by 8.

Set the numbers as in the margin, and proceed thus:—8 times 4 are 32; set down 2, and carry 3: 8 times 2 are 16, and 3 are 19; set down 9, and carry 1: 8 times 3 are 24, and 1 are 25; set down 25: and the product is two thousand, five hundred, and ninety-two, the same that would have been obtained by adding together eight lines, each 324: and thus it would be well to cause the pupil to work this and a few similar questions, to make him sensible of the identity of Multiplication and Addition.

(1)
Multiplicand... 324
Multiplier...... 8
Product... 2592

REASON OF THE RULE.

The principle on which this rule depends, is, that the products of the several parts of the multiplicand by the multiplier, or its parts, are, when taken together, equal to the product of the whole multiplicand by the whole multiplier. The reason for the mode of arranging the partial products, is, that figures of the same local value may stand in the same column. This will be understood from the following illustration.

In the example in the margin, we have first 7 times $7 = 49 = 4$ tens and 9; the 9 is set down and the 4 tens reserved. Then 7 times $9 = 63$, and 4 are 67, or 6 tens and 7. The 4 added to 63, is, by the nature of notation, equivalent to the 4 tens reserved, since 4 in the second place is equivalent to 40 in the first; 7 is then set down, and 6 tens, or properly 6 hundreds, reserved, and 6 added for them in the hundreds' place.

(2)
497
7
3479

The annexing of one cipher to a number expressed by the modern notation, makes its value 10 *times what it was before; two ciphers* 100 *times; three ciphers* 1000 *times,* &c. This is evident from the principles of notation, since the annexing of a cipher removes each figure one place to the left. Thus, the figure 7, with one cipher annexed, expresses 70, or 10 times 7; with two ciphers, 700, or 100 times 7, &c.

With the assistance of the preceding instructions, for practice multiply the following:

(3) 4596 by 2.
(4) 4708 by 3.
(5) 5681 by 4.
(6) 4983 by 5.
(7) 1297 by 6.
(8) 47965403 by 7.
(9) 85643279 by 8.
(10) 90038562 by 9.
(11) 47365491 by 10.
(12) 42306050 by 11.
(13) 4963756 by 12.

When the multiplier consists of several figures, as 346, 6749, &c. &c.

Rule. The multiplicand must be multiplied by each figure separately, (beginning with the right hand figure of the multiplier) and the first figure of every product must stand exactly *under the figure you multiply by.* Add these products together for the whole product.

Thus, multiply 6758 by 346.

(14)

$$\begin{array}{rl}
6758 & \textit{Multiplicand.} \\
346 & \textit{Multiplier.} \\
\hline
40548 & = 6758 \times 6 \\
270320 & = 6758 \times 40 \\
2027400 & = 6758 \times 300 \\
\hline
2338268 & = 6758 \times 346
\end{array}$$

$$\begin{array}{rl}
6758 & \textit{Multiplicand.} \\
346 & \textit{Multiplier.} \\
\hline
40548 & \\
27032 & \\
20274 & \\
\hline
2338268 & \textit{Product.} \\
\hline
\end{array}$$

From this the operation in the margin will be understood. The multiplier is equivalent to 300, 40, and 6, taken together; and, therefore, the multiplicand is multiplied by 6, 40, and 300, successively, and the sum of the products taken. In multiplying by 40, the product by 4 is taken, and a cipher annexed; and in multiplying by 300, two ciphers are annexed to the product by 3; in both instances according to the principle just delivered. In the second, or common form of the operation, these additional ciphers are omitted, and the omission is compensated by making the figures occupy the same places which they do when the ciphers are annexed; and hence the reason of setting the figures in the manner prescribed in the rule, is evident. When the pupil is learning the reason of the rule for Multiplication, it may be well to cause him for some time to annex the ciphers, in the mode above explained.

By the same method multiply the following:

(15) 4963 by 21.
(16) 4837 by 34.
(17) 497356 by 67.
(18) 8940507 by 98.
(19) 497436 by 365.
(20) 400076 by 883.
*(21) 999999 by 446.
(22) 470563 by 8432.
(23) 35142689 by 739.
(24) 86497320 by 8456.
(25) 92731128 by 97635.

When ciphers are intermixed with the figures in the multiplier.

Rule. Omit the ciphers, and let the first figure of each product be placed under its respective multiplier. Thus:

```
           (26)
  Multiply 4713457
       by    40806
        ----------
          28280742
        37707656
      18853828
      ------------
      192337226342
```

```
       (27)                         (28)
Multiply 4713457             Multiply 976382
      by 5704008                   by  60502
         -------                       -----
```

When there are ciphers at the end of the multiplicand or multiplier.

Rule. Neglect the ciphers, and multiply as before; then, to the right hand of the product, annex as many ciphers as were omitted. Thus:

```
         (29)
Multiply 4976 by 80.
           80
        -----
        39808
```

Having multiplied 4976 by 8, and produced 39808, you add a cipher, making the result 398080.

---

* There is a short method of arriving at the result of questions like the above, which will be explained in the proper place.

(30)

Multiply 459300 by 363000.

363000

13779
27558
13779

1667259 to this you must add *two* ciphers in the multiplicand, and *three* in the multiplier, making five; the product will therefore be 166725900000.

To multiply by two or more digits, in one line, commonly called the back-figure *method.*

This method is generally adopted when the left-hand figure of the multiplier is a unit, 13, 14, 15, 16, 17, 18, and 19. In which case the rule is as follows: Multiply the units' figure of the multiplicand by the right-hand digit of the multiplier, and of the product, set down the units' figure, retaining in your mind the left-hand figure to be carried. Multiply the *second* figure of the multiplicand—to the product add what were to be carried—and also the *first* figure of the multiplicand, which is called *taking in the back-figure.* Then set down the units' figure, and retain in your mind the number to be carried forward, as before. Multiply the *third* figure of the multiplicand, add the number to be carried, and also the *second* figure of the multiplicand, that being now the back-figure; and so on, to the end of the multiplicand. It may not be unnecessary, however, to remind the learner, that the *last* figure of the multiplicand must be taken in as a *back-figure,* and in order that it may not be omitted, place a dot on the left-hand side of the multiplicand, using the dot as the *representative* of another place; then the last figure of the multiplicand may, of course, be considered as the *back-figure* of that place.

EXAMPLE FOR ILLUSTRATION.

(31) Multiply 719386 by 13, in one line.

Operation.

·719386
13

9352018 *Proof.*

EXAMPLES FOR PRACTICE.

(32)	(33)	(34)
3265431	3292734	497632
13	14	15

(35)	(36)
423456	453682
16	17

To multiply any number of one denomination, by a composite number, of two parts.

First multiply the given number by one of the two component parts; and the result, by the other component part, and the last result will be the answer or product required.

(37)

Multiplicand	524367 × 36
	6
First result	3146202
	6
Product	18877212

EXAMPLES FOR PRACTICE.

(38)	(39)
47391526 × 14	38562047 × 45
7	5
2	9

METHODS OF PROOF.

Let the multiplier be taken as multiplicand, and the multiplicand as multiplier; and if the product thus obtained agree with the product found before, the process is right.

(40)

64	45
45	64
320	180
256	270
2880	2880

Thus, when 64 is multiplied by 45, the product is found to be 2880; and when 45 is multiplied by 64, the same result is obtained.

There is a method of correcting Multiplication by rejecting the *nines*, but as it is subject to error I do not give it; the foregoing method cannot fail to be correct, whilst it affords practice for the pupil.

Let the following, or such part as will be thought sufficient, be worked to impress the foregoing instructions.

(41) 94876843562
743
284630530686
379507374248
664137904934
70493494766566

(42) 4725963768
9846
28355782608
18903855072
37807710144
42533673912
46531839259728

(43) 27409400
1285000
1370470
2192752
3289128
35221079000000

(44) 8675764094
70906
52054584564
78081876846
60730348658
615163728849164

57610993 × 17, 19, 23
19446523 × 39, 67, 85
40567876 × 35, 59, 79
75408399 × 47, 58, 95
96025347 × 46, 82, 97
67128345 × 342, 675
38601576 × 563, 498
46739170 × 476, 586
28619976 × 723, 496
63147877 × 355, 627
36787582 × 806, 700
87594381 × 640, 697
70983569 × 7304, 681
286199768 × 654, 740
12345678 × 690, 790
312467136 × 784697
70280416 × 508, 804
31796080 × 790, 999
82619600 × 870, 708
79186907 × 6195843
49716396 × 1849000
50804500 × 4776050
65320190 × 8900764
91874963 × 9876543
81503600 × 7043000
27948019 × 7650992
58793880 × 8094730
62473894 × 2973586

DIVISION.

THIS rule instructs me to find how often any lesser number is contained in a greater number, or in other words, how often the lesser number can be taken out of or subtracted from the greater, hence, division is said to be a compendious subtraction, the result of which shows how often the lesser number is contained in the greater.

For example, suppose it was required to find how many times the number 4 can be taken from 24 by the rule of Subtraction, it will appear to be contained in it six times, and this number 6 expressing how often the 4 is contained in 24, is called the *quotient*, hence

In division there is given two numbers, the greater and the lesser, the former being the *divided* number, is called the DIVIDEND, and the latter being the *dividing* number, is called the DIVISOR, and the number required quoting how often the divisor is contained in the dividend, is called the QUOTIENT; that is, in the example before us, 4 is the *divisor*, 24 is the *dividend*, and 6 is the *quotient*, informing me that 4 is contained 6 times in the 24, and in this instance the 4 is said exactly to divide 24, therefore to measure it. But, were it required to find how often 4 was contained in 27, after taking 4 as often as possible out of it, it will appear not to be contained exactly in it as before, but to leave 3 remaining, that is, after taking 4 six times, or 24 out of 27, there will remain 3, hence, there appears another number in division, viz. a *remainder*, and,

Observe, *when a lesser number is not exactly contained in, or will measure a greater*, it will measure it best when after taking it from the greater as often as possible, it will leave a remainder less than the lesser number, or the divisor, for it is plain in the above instance, after taking the 4 six times out of 27, the remainder is 3, that is, less than 4, and notwithstanding there is the remainder 3, still 27 is said to contain 4 six times, but leaving a remainder of 3.

In DIVISION the given terms are placed after each other in one line, *the divisor on the left*, *the quotient on the right*, and *the dividend between them*, each separated by a stroke or circumflex. When the divisor and dividend are thus placed, and the place for the quotient on the right remaining blank, the principal difficulty will be to find the quotient figure or figures, for which purpose use the following

RULES *in their respective Cases.*

1. When the divisor is 12 or less, and will exactly measure the dividend. Consider from a recollection of the Multiplication Table what number multiplying the divisor will equal the dividend, that will be the quotient figure.

EXAMPLE.

(1)
Divisor. Dividend. Quotient.
3) 18 (6
18 Subtract.
—

In this Example recollect, 6 multiplying 3 = 18, the dividend, place this 6 in the quotient, then having multiplied the divisor, 3, by 6, place the product 18, under the dividend 18, and subtracting this 18 from the dividend, and finding nothing remain, 6 is the true quotient, and informs me that 3 can be taken out of 18 exactly 6 times.

In the same manner proceed with the following Examples:

(2) 4)28((3) 5)35((4) 6)48((5) 8)64((6) 7)72(

(7) 7)48((8) 9)72((9) 11)121((10) 12)96(

For proof in this case, if your quotients be right, the quotient multiplying the divisor must equal the dividend.

2. When the divisor is 12 or less, but not contained in the dividend exactly as before. Consider from the Multiplication Table, what number multiplying the divisor will give a product next less than the dividend, that will be the quotient, and this quotient when its product as before is taken from the dividend (if right) must leave a remainder less than the divisor, as for Example:

(11)
Divisor. Dividend. Quotient.
5) 39 (7
35 Subtract.
4 *Remainder.*

Here, from the Table, recollect that 7 times 5 gives 35, the next less product than 39, for 8 times, or 40, is too much, and 6 times, or 30, is too little, therefore I place 7 in the quotient, then as before, multiplying 5 by 7, I place the product 35 under the dividend 39, and subtracting 35 from 39, I find 4 (less than 5) remaining, hence my quotient, 7, is right. But to prove this example, multiply the quotient 7, by the divisor 5, and to the product add the remainder 4; if the sum be equal the dividend, the quotient is right; thus, 7 × 5 = 35, and 35 + 4, the remainder, = 39, which being equal to the dividend, the quotient is right.

In the same manner proceed with the following Examples:

(12) 4)25(
(13) 5)38(
(14) 6)47(
(15) 8)46(
(16) 9)65(
(17) 7)65(

(18) 11)27(
(19) 12)49(
(20) 11)63(
(21) 12)87(
(22) 9)104(

3. When the divisor is 12 or less, and the dividend of many figures, not contained in the table. In this case, the quotient must be found by several steps or divisions, thus, beginning with the leading figure or figures of the dividend as a first dividend, find the quotient and remainder, if any, by either of the former examples or rules; to the remainder, if any, bring down the next figure of the dividend, at the same time placing a dot under it *to signify it is brought down;* this figure, with the remainder, becomes a new or second dividend, then dividing as before, and placing each quotient figure as found in the quotient after each other; in this manner, when all the figures of the dividend are brought down, the sum is done. And to prove it, multiply the quotient by the divisor, and to the product as before, add the last remainder, if any; this sum must equal the dividend if the work be right.

EXAMPLE.

```
              (23)
Divisor.  Dividend.  Quotient.
  8)      3476028 (434503
 8×4=32 · · · · ·        8
             ——————————————
           Proof 3476024
        27 new dividend
  8×3=24
        36 new dividend.
  8×4=32
        40new dividend.
  8×5=40
        28 new divid.
  8×3=24
Remainder  4
```

Here finding at first the divisor 8 is not contained in 3, the leading figure of my dividend, I divide 8 into 34 the two leading figures, and by 2nd rule I find my quotient to be 4, which I place in the quotient, and proceeding by 2nd rule, I find my remainder 2 less than my divisor, to which I bring down 7, the next figure of my dividend, placing a dot under it, this gives me with the remainder 2, 27, which is now a new dividend, and dividing this 27 by 8, my divisor as before, per rule 2nd, I find my quotient 3, and, after multiplying and subtracting I find my remainder to be 3, and being less than my divisor, the quotient is right, to which I bring down 6, the next figure of my dividend, this gives a new dividend of 36 which I again divide by 8 as before, and find my quotient to be 4, and again multiplying 8, my divisor, by it, I find my remainder to be 4, to which bringing down the next figure 0, my new dividend is 40; on dividing this, I find by the first rule my quotient is 5, and the remainder nothing, then bringing down the next figure 2, this is now my new dividend, and here perceiving that 8 my divisor is not contained in the 2, I put a cipher in my quotient, signifying that 8 is not contained in it, and lastly to this 2 I bring down my last figure 8, which gives me the last dividend 28, and lastly, dividing this as before, per rule 2nd, I find my last quotient to be 3, and the remainder 4, less than 8, and having brought down every figure in the dividend I find my quotient to be 434503 and 4 remaining, that is, 8 is contained so many times in 3476028 and leaves 4. But,

This work can be done much shorter by placin quotient under the dividend, and performing the intermediate steps in the mind as follows:

8)3476028

434502 and 4 rem.

Here, say 8 into 34, 4 times, I place 4 under the 4 of 34 and 2 remains, this 2 out of the 4 with the next figure 7, but, placed before it, will make 27, then say 8 into 27 3 times, which 3 put under the 7 and 3 remains, which 3 remaining out of the 7, with the next figure, 6, will make 36, then say 8 into this 36 goes 4 times, and leaves 4, put the quotient 4 under the 6, and the 4 remaining with the 0 as before make 40; then go on and say 8 into 40 is 5 times, but having no remainder place this 5 under the 0 and say 8 into 2, 0 or no times, which 0 place under the 2, this 2 now remaining with the next figure, 8, will make 28, then say 8 into 28 is 3 times, and leaves 4, then placing 2 under the 8, the quotient is as before, 434508 and 4 remaining.

From what has been explained, work the following, and to assist and encourage the Pupil, I have put some figures in the working to show when the work is correctly performed.

(24) Divide 4976582 by 3.

Dividend.
Divisor 3)4976582
·6·8··0 *Remainder* 2.

Cases of this kind where the *quotient is placed under the* divisor is termed *short* Division.

Divide 49762000 by 5.
0·9··4·0

(25) Divide)58763258 by 4.
··6·0·1· *Remainder* 2.

	Divisors.	Dividends.		Divisors.	Dividends.
(26)	6	66978324.	(34)	14	50607080905.
(27)	7	5680907654.	(35)	15	4560759011.
(28)	8	3256873950.	(36)	16	3264168096.
(29)	9	687432961.	(37)	17	345115380.
(30)	10	109658732.	(38)	18	294832681.
(31)	11	880192345.	(39)	19	87326584.
(32)	12	972602436.	(40)	20	93672684.
(33)	13	876932865.			

Again, *when the divisor as well as the dividend consists of many figures*, the principal difficulty will be to find the quotient figure, for which purpose observe the following

Rule. First count from the leading figures of the dividend as many as will make a number sufficient to contain your divisor, *which can never exceed it but by one figure*, call these figures your first dividend; Secondly, inquire how often the leading figure of the divisor is contained in the leading figure or figures of this dividend, *which, take as often as you can, but always to leave a remainder;* Thirdly, consider this remainder as so many tens before the next figure of the dividend, then try if the second or next leading figure of the divisor go as often into this number as the first did, *if it should*, and leave a remainder more than, or nearly equal to the quotient found, *this quotient figure thus found will be right, but*, if the second figure will not be as often contained as the first was found to be, then the first quotient must be reduced one or two less *(keeping it still as much as possible)* until the second figure, &c. can be as often contained, *now*, having thus found your quotient figure, proceed with it as in the former example, always seeing that the remainders are less than the divisor, before you bring down any figure of the dividend, and thus proceed until all the figures are brought down, when the work will be finished.

EXAMPLE.

Seeing that 3, the leading figure of the divisor, is greater than 2, the leading figure of the dividend, and as my divisor has four figures in it, my first dividend must therefore consist of five, that is 22453, then I say 3 into 22 the leading figure as often as I can, that is, 7 times, &c. which leaves 1, this 1 before the next figure 4 of my dividend is only 14, but the second figure 5, of the divisor not going 7 times into

```
             (41)
3567)22453915679(6294902
     21402······
     -----
      10519 new dividend.
       7134
       -----
       33851 new dividend.
       32103
       -----
        17485 new divid.
        14268
        -----
         32176
         32103
         -----
            7379
            7134
            ----
             245 remaind.
             ---
```

this 14, therefore 3 cannot be taken 7 times into 22, but only 6 times, again, trying 3, 6 times, in the same manner into 22, it leaves 4, which with the next figure 4 of the dividend gives 44, into which finding 5 the next figure of my divisor will go 6 times, and leave a sufficient remainder being greater than 6, my quotient 6 is right, then, placing 6 in the quotient, I multiply the divisor by 6, and put the product 21402 under the dividend, and by subtracting I find the remainder to be 1051, which being less than the divisor the quotient is right, to this bring down the next figure 9 of the dividend and it makes my next or new dividend to be 10519. In this new dividend I find the divisor is contained 2 times, which I place in the quotient as before, then multiplying the divisor by 2, and placing the product 7134 under this dividend, and by subtracting it as before, the remainder is 3385, this again being less than the divisor, the quotient is right, I then bring down the next figure 1, and my new dividend is 33851, then proceeding in like manner from this dividend, until every figure is brought down, as in the work, until I find the remainder to be only 73, then bringing down the 7, the dividend will be only 737, here the divisor having four figures will not be contained in this number having only three; I therefore put a 0 in the quotient, and bring down the next and last figure 9 of the dividend, this makes the number or last dividend to be divided 7379, in which my divisor going 2 times or twice, I put 2 in the quotient, and after multiplying, &c. as before, the quotient is found to be 6294902, and 245 remains.

Division can be easily proved again by addition, thus, by adding the remainder, if any, to the several products of the divisor by the quotient figures as they stand under each other in the work; if their sum be equal to the dividend, the work will be right. See the last example from which the following products are taken and placed under each other as they stand in the example, which see.

These added will be equal to

21402
7134
32103
14268
32103
7134
245 Remainder.

And these products might be added (in the work without taking them out.)

the divid. 22453915679 See the example.

But when you intend to prove by addition, great care must be taken to keep the figures exactly under each other, otherwise it will be impossible to add them as they stand in the work when so placed, but if rightly placed to take them out this way would be unnecessary.

EXAMPLES.

(42)
36)26578670(
here there will remain 14

(43)
49)356742(
remainder 22

(44)
58)3670902(
remainder 24

(45)
467)26578670(
remainder 183

(46)
534)6739801(
remainder 187

(47)
608)5206718(
remainder 416

(48)
5674)246753016(
remainder 2104

(49)
68045)426704389(
remainder 62239

When your divisor has one, two or, more, ciphers in the lowest places to the right hand, cut them off by a comma or stroke, and in like manner cut off or separate the same number of figures from the dividend, the remaining figures being divided as before by the significant figures of the divisor, the quotient will be the true quotient, but to the remainder, if any, by annexing the figures cut off from the dividend, you find the true remainder.

EXAMPLES.

(50)

3564,00)27604878,26(

here the true remainder will be 169826

(51)

36508,0)49067543,2(

the remainder is 79122

Should the divisor be a unit with one or more ciphers annexed to it, as 10; 100; 1000; &c, in this case, point off with a comma from the dividend as many figures as there are ciphers from the right hand, and the figures remaining, on the left will be the quotient required, and those cut off the remainder.

EXAMPLES.

(52) 24365 divided by 10, the quotient is 2426,5 remainder as pointed off is 5.

(53) 24365 divided by 100 the quotient is 243,65 remainder as pointed off is 65.

(54) 24365 divided by 1000 the quotient is 24,365 remainder as pointed off is 365, &c.

Division is always proved best by multiplication, that is, to multiply the divisor and quotient together as before shown, their product with the remainder added (if any) will equal the dividend, thus $48 \div 9 = 5$, and 3 remains, then $5 \times 9 + 3$ the remainder$=48$, the dividend, &c.

I have observed some authors have mentioned, that division can be proved by dividing the dividend by the quotient, and the quotient thus found will be equal to the divisor, and leave the same remainder, but this will only hold good when the remainder is less than the quotient, for it must be evident, if the remainder of any sum be equal to or more than the quotient, which may be the case when the divisor has more figures than the quotient, the example will not prove, for the quotient in that case must be contained oftener in the dividend than the divisor expresses; thus, in the foregoing examples, where 68045 is the divisor, you will find the remainder is 62239, add the quotient only 6270, then, making 6270 the divisor, the new quotient will be 68054, greater than 68045, and leave 5809 remaining, not 62239 as before, therefore division cannot be proved generally by division, but only when the remainder is less than the quotient, in such a case I would then recommend division in preference as a proof, were it only for exercising the rule, and here I must observe,

Division can be contracted, First, when the divisor is a single figure, and the dividend many figures. This was illustrated before in the example, where 3476023 was divided by 8, to which I refer.

Secondly. When the divisor is any of the composite numbers in the multiplication table, it can be shortened by the following

Rule. Divide first by one component part, as in the first instance above referred to, placing the quotient under the dividend, reserving the remainder, or placing it apart on the right, and then by dividing this quotient again, in the same manner by the other component part, placing the quotient under the former, this quotient last found will be the quotient required. But to find the true remainder you must multiply the first divisor by the last remainder, and to the product add the first remainder, if any, the sum will be the true remainder. But should nothing remain in the last division, the first remainder will be the true remainder again, should a remainder happen to be in the last division, and no remainder in the first, then the product of this last remainder into the first divisor will be the true remainder.

EXAMPLE.

(55)

42)563427 here 7×6=42, therefore divide by 6
——— and by 7.
again 7)93904($\frac{3}{6}$)=first quotient by 6 (kept in mind)
——— and 3 remains written thus $\frac{3}{6}$.
this 13414 (6) = second quotient by 7 and 6 re-
——— mains, then the true quotient is 13414,
and (6×6+3)=39 is the remainder, or again for proof,
42)563427 I here divide by 42, first by 7, and then
——— by 6, this must prove the above.
6)80489 ($\frac{4}{7}$) = quotient by 7 and 4 remains.
———
13414 (5) = quotient by 42 and 39, (that is 5×7
——— +4), remains as before.

EXAMPLES FOR ILLUSTRATION.

(56)
35)463275
remainder 15

(57)
56)385746
remainder 18

(58)
72)365472
remainder 56

(59)
84)3675097
remainder 13

(60)
48(376054
rem. 22

(61)
63)427590
remainder 9

(62)
96)430276
remainder 4

(63)
121)3760842
remainder 41

Thirdly. *When the divisor is many figures*, the work will be much shortened by leaving out the products of each quotient figure, which may be done by subtracting, as you multiply each product from the figures of the dividend, observing to increase each figure by as many tens as may be necessary to take the product from it, and by putting down only the several remainders as you multiply and subtract, many figures will be saved; but after putting down each difference or remainder, be careful to carry as many tens to the next product as you increased the figures of the dividend. See the following

EXAMPLE.

Divide 350459806 by 4763.

(64)
4763)350459806(73579
17049
27608
37930
45896
3029 remainder.

Perceiving the leading figure 4 of my divisor is not contained in 3 the leading figure of the dividend, and my divisor containing four figures, I draw a line under the first five figures, for my first dividend thus 35045, then finding my first quotient is 7, I say 7 × 3 are 21, this being to be taken from 5 I increase it 2 tens and call it 25, then 21 from 25 and 4 remain, which place under the 5 and carry 2, for the two tens added to 5;

again say, 7 × 6 are 42, and 2 carried are 44, this must be taken from 44, increasing the 4 four tens, and leaving no remainder, put down 0 under the 4 and carry 4 for the 4 tens added, again, 7 × 7 are 49 and 4 carried are 53, this taken from 60 (that is increasing the 0, 6 tens) leaves 7, which place under the 0 and carry 6; lastly, 7 × 4 are 28 and 6 carried are 34, which taken from 35 leaves 1, this being put under the 5 of 35, you have the first remainder, viz. 1704, and being less than the divisor, is right, to this remainder bring down the next figure 9 of the dividend, your new dividend is 17049, into which the divisor will go 3 times, then placing 3 in the quotient proceed in the remaining steps in a similar manner.

QUESTIONS TO EXERCISE.

	Divisors.	Dividends.		Divisors.	Dividends.
(65)	21	4236837654.	(74)	83	420156132.
(66)	22	8749658326.	(75)	126	873649820.
(67)	23	4069874284.	(76)	197	49658349.
(68)	34	5567834268.	(77)	284	47368265.
(69)	45	8741219087.	(78)	565	4762834.
(70)	67	123456789.	(79)	4976	8376294.
(71)	84	987654321.	(80)	8342	987654321987.
(72)	42	100000000.	(81)	48765	832649568740.
(73)	65	843265976.	(82)	678245	768436958430

COMPOUND RULES.

Having given, I trust, sufficient information on the simple Rules, which I hope the Pupil has got acquainted with, the next thing to be obtained is a knowledge of the *Compound Rules*, and to acquire facility on that subject, the pupil will find it necessary to become well acquainted with the Arithmetical Tables, page 5, but particularly with the pence and shilling tables, as the nature of the instruction following will prove. I also here insert a copy of the Pence Table, differing a little from that on page 6.

PENCE TABLE.

		s.	d.			s.	d.
12	pence are	1	0	70	pence are	5	10
13	....	1	1	72	....	6	0
14	....	1	2	78	....	6	6
15	....	1	3	80	....	6	8
16	....	1	4	84	....	7	0
17	....	1	5	90	....	7	6
18	....	1	6	96	....	8	0
19	....	1	7	100	....	8	4
20	....	1	8	108	....	9	0
24	....	2	0	110	....	9	2
30	....	2	6	120	....	10	0
36	....	3	0	130	....	10	10
40	....	3	4	132	....	11	0
48	....	4	0	138	....	11	6
50	....	4	2	144	....	12	0
60	....	5	0	150	....	*12	6
65	....	5	5				

MONEY.

4 farthings. qrs. make 1 penny. *d.*
12 pence..................... 1 shilling. *s.*
5 shillings.................. 1 crown. *cr.*
20 shillings 1 pound or sovereign. *£.*
21 shillings 1 guinea. .... *G.*

* This Table, when found difficult for a child to commit to memory, I never used until he advanced; but, in its place, I made him use the Multiplication Table of twelves, by which he may immediately discover how many shillings were in any number of pence, under 150, which is quite sufficient for his first examples, as he proceeds, he himself will discover the necessity of using the above table for greater expedition, when he may occasionally commit it to memory.

COMPOUND ADDITION.

In ADDITION OF MONEY, *observe the following* RULE.

Place the several sums to be added under each other; pounds under pounds, shillings under shillings, and pence under pence, &c.; then, add up the lowest denomination, in the total of which, find from the table, how many of the next denomination are contained in it, and what, (or how many of it) may remain: this remainder put under its column, and carry to the next column so many of it as you found in the total of the column you added, *in this manner* proceed with the farthings and pence; but when you add the column of shillings, first, add the shillings found in the pence column to the units of the shillings, as you would, were they whole numbers; putting down any odd shillings over the tens found therein: then carrying the tens to the tens of the shillings, add up the tens, and if the tens found be odd, place the odd ten by the shillings before put down, and carry half the remaining even number of tens to the unit's column of pounds; which, with the other columns of the pounds, add up as you would whole numbers, (for as two tens, or two half pounds, make one pound) therefore, half the tens of shillings, are added as so many pounds.

EXAMPLE.

(1)

£.	*s.*	*d.*
25	14	6
27	6	10
54	15	4
20	17	9
128	14	5

Add £25..14..6. £27..6..10. £54..15..4. and £20..17..9. together. Place them as in the margin; then, adding the pence, say, 13, 23, 29, that is, 29 pence, which is 2*s.* 5*d.*; put down the odd 5 under the pence column, and carry the 2 shillings to the units of the shilling's column, and say, 9, 14, 20, 4; put down the odd 4 under the units of shillings, and carry the two tens to the tens of the shillings, which, with the number of tens there, will make five tens; these being odd, I put the odd ten by the 4 shillings (before put down,) making it 14 shillings; then I carry half the remaining 4 tens (making two pounds,) to the pounds, and adding the pounds as in addition of whole numbers, I find the pounds make £128., and the total sum required is £128..14..5.

The pence column might be added up thus, with a little practice, by making shillings of the pence as at each step you add; thus, seeing *4d.* and *9d.* make *13d.*, at *4d.* say *1s. 1d.*, at *10d.* say *1s. 11d.*, and at the *6d.* say *2s. 5d.*, thus converting the totals at every step into shillings and pence; when you come to the top, you have only to put down the odd pence, and immediately add the shillings. This method, if not approved of, has at least the advantage of novelty, and is curious. I have tried it often with success.

Either of the two first methods of proof given in Simple Addition, may be employed in Compound Addition.

In this example, the sum of the farthings is 14, which being divided by 4, the farthings in a penny, the quotient is *3d.*, and the remainder, 2 farthings, or a halfpenny, is set down. The quotient, 3, is then added with the pence; the sum, 54, being divided by 12, the pence in a shilling, the quotient is 4 shillings, with a remainder of 6 pence, which is set down. The quotient, 4, is then added with the units of the shillings; the sum is 39, of which the last figure is set down, and the tens being carried to the tens of the shillings, the sum is 8, which being divided by 2, because 2 tens, or 20 shillings, make a pound, the quotient is 4, which is added with the pounds, as in Simple Addition.

(2)

£.	*s.*	*d.*
39	18	7¾
51	12	4½
79	19	10½
8	7	11¼
43	13	9¾
375	16	10¾
599	9	6½

In this example the halfpence amount to 5, or 2½*d.* In such examples, where all the fractional parts are halfpence, it is easier to call each a halfpenny, than two farthings.

(3)

£.	*s.*	*d.*
15	3	8½
31	16	7½
94	13	8½
55	12	11½
37	11	9½
234	18	9½

FURTHER EXAMPLES.

(4)

£.	s.	d.
36	12	4
35	6	8½
47	19	5¼
34	14	10¼

(5)

£.	s.	d.
36	18	4½
25	12	8¼
63	15	10¾
74	9	11½

(6)

£.	s.	d.
37	16	3¼
45	12	2½
64	19	5¼
49	4	11

(7)

£.	s.	d.
32	17	6½
47	15	4¼
38	10	0¾
61	17	3¼

(8)

£.	s.	d.
36	19	5½
65	15	8¼
84	10	2
45	9	0½
71	7	7¾

(9)

£.	s.	d.
231	15	4½
365	12	5¼
400	14	0½
321	9	10
47	11	6¾

(10)

£.	s.	d.
460	12	3½
221	14	9¼
403	8	10
126	17	6¾
257	15	4¾

(11)

£.	s	d.
365	19	5½
304	15	10¼
603	14	0½
215	11	11¾
140	9	9½

(12)

£.	s.	d.
320	8	10½
445	15	11¾
360	17	9¾
475	14	0¼
832	19	7½
777	2	1½
207	10	9¾

(13)

£.	s.	d.
2546	12	8¼
1367	3	3
456	13	9¾
753	12	8½
657	10	5¼
2635	19	0
120	4	8½

(14)

£.	s.	d.
1365	15	2½
7420	16	4¼
1575	9	2½
267	10	10¾
468	15	4
1365	19	3½
407	10	11¼

(15)

£.	s.	d.
3632	17	4¼
166	10	10
802	9	7½
1530	11	1¼
421	19	11¼
826	14	7¼
800	19	2
3385	15	7½

(16)

£.	s.	d.
1675	13	8½
260	5	1
20	19	11¼
2641	11	3¾
630	15	2
601	15	10½
1924	16	8½
467	13	8¾

(17)

£.	s.	d.
3602	12	8¼
1562	14	3
365	15	4½
42	12	8¾
632	5	4½
7300	10	11
532	18	6½
2075	2	10

(18)

£.	s.	d.
4630	12	8¼
365	14	2
676	15	6
3064	18	11¼
360	17	7
807	18	4¼
36	19	1
410	10	2¼
542	9	11
421	10	8½
2773	19	0¼

(19)

£.	s.	d.
463	16	2¼
370	18	10
36	14	11½
407	16	3¼
26	15	2¾
360	10	0½
419	9	11½
604	13	10¼
560	14	8¼
174	8	10
2077	17	7¼

(20)

£.	s.	d.
3256	12	4¼
250	9	8
587	19	3½
27	15	8
376	0	10
938	18	0
1467	17	9¼
376	15	10
275	14	11¾
1509	0	9
28	13	3¼

(21)

£.	s.	d.
267	12	9¼
134	19	2
67	14	3½
371	18	9¾
27	12	10
3409	10	11½
280	19	4
375	8	8¾
406	7	11
257	18	3
9074	17	9½
737	19	5¼

(22)

£.	s.	d.
274	16	5¼
39	12	8
274	16	5½
3095	8	11
297	17	7½
59	8	9¾
374	6	8
1509	12	10½
308	4	6
532	18	0¾
4096	12	3
8409	0	11½

TROY WEIGHT.

Gold, Silver, and Jewels, are weighed by this Table.

4 grains	1 carat.
24 grains or 6 carats	1 pennyweight.
20 pennyweights	1 ounce.
12 ounces	1 pound.

OF TROY WEIGHT.

In general, to avoid a multiplicity of rules in adding these and other weights or measures, use the following General Rule.

Having placed the several denominations under each other, as before, in money, then add up the lowest denomination, and divide its amount by so many of the column just added, as are contained in one of the next columns, and if any remains after division, put it down under the column, and carry the quotient to the next column, where proceeding in the same manner with each column, except the highest, which must be added without division, the several remainders under each column, with the sum under the highest, will be the sum required.

EXAMPLES.

(1)			(2)			(3)			(4)		
oz.	*dwt.*	*grs.*	*oz.*	*dwt.*	*grs.*	*oz.*	*dwt.*	*grs.*	*oz.*	*dwt.*	*grs.*
25	13	21	34	17	11	61	16	23	46	15	21
16	12	20	24	16	20	34	10	19	37	12	19
16	10	14	17	15	23	47	9	16	45	16	12
23	12	22	20	4	19	20	14	7	83	9	19
82	10	5									

AVOIRDUPOIS WEIGHT.

Bread, Groceries, and all coarse articles, are weighed by this Table.

16 drams..........................1 ounce.
16 ounces ....1 pound.
14 pounds ....1 stone.
28 pounds ....1 quarter.
4 quarters or 112 pounds .....1 hundredweight.
20 hundredweights ....1 ton.

144 pounds Avoirdupois, are equal to 175 pounds Troy.

OF AVOIRDUPOIS WEIGHT.

In this example the sums of the pounds, quarters, and hundreds, are respectively divided by 28, 4, and 20, the remainders set below their respective columns, and the quotients carried to the next columns, respectively. The hundreds may be added as shillings in Addition of Money, the divisor being the same in both cases.

(1)			
t.	*cwt.*	*qr.*	*lb.*
35	16	0	20
42	14	2	18
18	9	1	16
17	18	3	7
31	5	3	19
45	12	2	
191	17		

(2)			(3)			(4)			(5)			
cwt.	*qr.*	*lb.*	*cwt.*	*qr.*	*lb.*	*cwt.*	*qr.*	*lb.*	*t.*	*cwt.*	*qr.*	*lb.*
35	2	14	46	2	15	46	2	21	15	17	2	21
29	1	16	37	1	25	32	1	16	12	18	1	16
40	3	25	10	3	24	26	2	20	24	9	0	13
18	0	16	16	0	21	30	0	14	61	14	1	27
12	3	27	18	1	0	27	2	27	64	19	0	20
13	2	16	27	2	26	34	1	19	19	12	3	18

In adding the column of pounds, the division by 28 will be tedious and take up time; to avoid this, the following table of quarters, if committed to memory, will be very useful, and save trouble and time, viz.

28 pounds	1 quarter.
56	2 quarters.
84	3 quarters.
112	4 quarters.
140	5 quarters.
168	6 quarters.
196	7 quarters.

This Table being well known, any number of pounds can be made into quarters very easy—thus,
156 *lbs.* = 5 *qrs.* 16 *lbs.*
199 *lbs.* = 7 *qrs.* 3 *lbs.*
148 *lbs.* = 5 *qrs.* 8 *lbs.*

Apothecaries' Weight being only used in compounding their medicines, and as examples of the addition, or compounding, &c. of the parts never occur in trade, they are here omitted.

LONG MEASURE.

3 barley corns	1 inch.
4 inches	1 hand.
12 inches	1 foot.
3 feet	1 yard.
6 feet	1 fathom.
5½ yards, or 16½ feet	1 rod or pole.
40 rods or poles	1 furlong.
8 furlongs, or 1760 yards	1 mile.
3 miles	1 league.
69½ miles	1 degree.

(1)			
per.	*yds.*	*ft.*	*in.*
14	5	2	1
39	6	1	11
32	1	1	8
19	0	2	6
28	5	1	7
31	6	2	11

(2)						
mls.	*fur.*	*per.*	*yds.*	*ft.*	*in.*	*bc.*
14	7	21	6	1	11	1
1	6	31	4	1	8	2
61	3	36	3	2	9	2
129	2	0	2	0	6	0
5	5	12	1	2	0	1
4	0	27	5	1	4	2

DRY MEASURE.

2 pints	1 quart.
4 quarts	1 gallon.
2 gallons	1 peck.
4 pecks	1 bushel.
8 bushels	1 quarter.
36 bushels	1 chaldron.

(1)

pks.	*gal.*	*qts.*	*pts.*
32	1	3	1
64	1	2	0
21	0	1	1
54	1	0	1
46	1	3	0

(2)

qrs.	*bar.*	*bu.*	*pks.*	*gal.*	*qts.*
3	1	3	2	1	5
17	0	1	2	1	3
14	1	2	1	0	6
15	1	0	0	1	0
9	0	3	3	1	7

WINE MEASURE.

2 pints (*pts.*)	1 quart.
4 quarts	1 gallon.
10 gallons	1 anker.
18 gallons	1 rundlet.
31½ gallons	1 barrel.
42 gallons	1 tierce.
63 gallons	1 hogshead.
84 gallons	1 puncheon.
2 hogsheads	1 pipe or butt.
2 pipes	1 tun.

SOLID OR CUBIC MEASURE.

1728 cubic inches	1 cubic foot.
27 cubic feet	1 cubic yard.
40 feet of rough timber or 50 feet of hewn ditto	1 load or ton.
42 feet	1 ton of shipping.
277¼ cubic inches	1 gallon.

ALE, BEER, AND PORTER MEASURE.

2 pints (*pt.*)	1 quart.
4 quarts	1 gallon.
9 gallons	1 firkin.
2 firkins	1 kilderkin.
4 firkins	1 barrel.
54 gallons	1 hogshead.
2 hogsheads	1 butt.
2 butts	1 tun.

WOOL WEIGHT.

14 pounds	1 stone.
2 stones, or 28 pounds	1 tod.
6½ tods, or 13 stones	1 wey.
2 weys	1 sack.
12 sacks	1 last.
12 score, or 240 pounds	1 pack.

CLOTH MEASURE.

2¼ inches	1 nail.
4 nails	1 quarter.
3 quarters	1 Flemish ell.
4 quarters	1 yard.
5 quarters	1 English ell.
6 quarters	1 French ell.

TIME.

60 thirds (*th.*)	1 second.
60 seconds	1 minute.
60 minutes	1 hour.
24 hours	1 day.
7 days	1 week.
4 weeks	1 month.
365¼ days, or 8766 hours	1 Julian year.

(1)

w.	*d.*	*h.*	*m.*	*sec.*
32	2	21	34	46
15	5	19	21	31
16	4	12	46	21
21	0	20	53	50
34	6	4	22	22

IN SUBTRACTION OF DIVERS DENOMINATIONS,

Nothing further is necessary than to direct that each denomination may stand under that of the same kind; and, when placed, compare the quantities of the greatest denomination, in order to distinguish the lesser quantities to be subtracted, and, when you know the lesser, in taking one denomination from its kind, if the quantity to be subtracted be greater than that from which it is to be taken, always increase the lesser by one taken from the next higher, and, when the difference is then found, put it down under it, and carry one to the next quantity to be subtracted, and in that manner proceed until you subtract the highest denomination.

EXAMPLE.

Here £28. being less than £36. I must take £28..10..8¼. from £36..14..5½. thus, one farthing from one halfpenny and one farthing remains, which farthing I place under the column of farthings; again, observing 5 pence is less than 8 pence, I increase the 5 pence by 12 pence, (the pence in one shilling taken from the 14 shillings) this makes 17 pence; then 8 from 17, 9 pence remains, which I place under the pence, and carry the 1 shilling taken from the 14 to the 10 in the lesser number, it makes 11, which taken from 14, leaves 3 shillings, which 3 I place under the shillings, and subtract the pounds, as in whole numbers, and the difference between the given sums is £8..3..9¼.; or, if done by addition, the work will stand thus much shorter, ¼ and ¼ make ½; 8*d.* and 9*d.* make 1*s.* 5*d.* and go 1; 11 and 3 make 14; 8 and 8 make 16 and go 1; 3, &c. therefore, in a few words, the difference is £8..3..9¼. To prove this, I subtract, in like manner, this difference again from £36..14..5½. the greater sum, and finding it leaves £28..10..8¼. = the lesser, the difference found is right; or, it may be proved otherwise, by adding £8..3..9¼. to the lesser sum £28..10..8¼. and as it makes £36..14..5½. = the greater sum, the work is right.

(1)

	£.	*s.*	*d.*	
from	36	14	5½	
take	28	10	8¼	
	8	3	9¼	diff.
	28	10	8¼	lessr.
	36	14	5½	gr.

In this manner may all subtraction of divers denominations of what nature soever, be done and proved.

(2) From £33..17..10¾
take £18.. 8.. 4¼.

£.	*s.*	*d.*	
33	17	10¾	
18	8	4¼	
15	9	6½	*Ar*

(2) Required the difference between £159..9..4¼ and £86..17..8½.

In this example, as a halfpenny is greater than a farthing, it is taken from a penny, and the remainder being added to the farthing, the sum, three farthings, is set down; a penny is then carried to 8 pence, and the sum being taken from one shilling, the remainder 3 is added to 4 pence, and the amount set down. We then proceed thus: 1 and 7 are 8; 8 from 9 and 1 remains; 1 from 2, (the tens' figure in the shillings in a pound) and 1 remains; 1 and 6 are 7; 7 from 9 and 2 remain, &c.

(3) £.	s.	d.
159	9	4¼
86	17	8½
72	11	7¾

EXAMPLES IN MONEY.

Find the difference between the following sums, and prove each both ways.

	(4) £	s.	d.	(5) £.	s.	d.	(6) £.	s.	d.
	346	10	8	34	12	6½	38	12	7¼
	129	14	2	47	10	3	13	18	9½
diff.									
less.									
great.									

		£.	s.	d.		£.	s.	d.
	(7)	46	12	6	(8)	62	12	3½
		94	15	8½		81	19	0¼
diff.								
less.								
great.								

		£.	s.	d.		£.	s.	d.
(9)	lent	34	12	6	received	46	10	2¼
	more	70	16	3½	more	6	12	8¼
	more	65	10	4¼	more	91	18	11¼
	more	16	15	3	more	6	16	6
	more	4	10	7¾	more	1	12	7½

What remains of these due or unpaid?

I have in some cases put the greater sum under, though different from the usual custom, that the pupil may place them in the correct position.

	£.	s.	d.		£.	s.	d.
(10) *Dr.*	461	12	8¼	*Cr.*	219	15	8½
	210	10	11		210	15	6
	47	18	3¼		100	19	5¾
	148	16	10		96	19	11½
	300	10	8½		264	8	8¼

What is the balance of this account?

TROY WEIGHT.

	lb.	*oz.*	*dwts.*	*grs.*
(1)	35	9	15	12
	14	3	17	16
diff.				
lessr.				
gr.				

	oz.	*dwts.*	*grs.*
(2)	16	13	12
	8	15	13
diff.			
lessr.			
gr.			

	oz.	*dwts.*	*grs.*
(3)	21	10	19
	19	15	22
diff.			
less.			
gr.			

	oz.	*dwts.*	*grs.*
(4)	31	16	20
	42	12	19
diff.			
less.			
gr.			

AVOIRDUPOIS WEIGHT.

	cwt.	*qrs.*	*lb.*
(1)	16	2	21
	26	3	24

	cwt.	*qrs.*	*lb.*
(2)	82	1	19
	12	2	22

	t.	*cwt.*	*qrs.*	*st.*	*lbs.*	*oz.*	*dms.*
(3)	14	13	1	1	12	14	10
	10	15	2	0	13	12	15
diff.							
lessr.							
gr.							

LONG MEASURE.

		yds.	*ft.*	*in.*		*per.*	*yds.*	*ft.*	*inch.*
	(1)	34	2	10	(2)	37	4	2	5
		41	2	9		28	6	1	10
diff.									
less.									
gr.									

DRY MEASURE.

		bush.	*peck.*	*gals.*		*qrs.*	*bush.*	*peck.*
	(1)	42	3	2	(2)	17	1	0
		31	2	3		12	1	1
diff.								
less.								
gr.								

LIQUID MEASURE.

		t.	*hds.*	*gals.*		*hds.*	*gals.*	*pts.*
	(1)	21	1	32	(2)	64	42	7
		12	2	41		79	44	5
diff.								
less.								
gr.								

LAND MEASURE.

		a.	*r.*	*p.*		*a.*	*r.*	*p.*
	(1)	31	9	57	(2)	41	3	21
		24	3	12		73	2	37
diff.								
less.								
gr.								

COMMON TIME.

	yrs.	*w.*	*d.*	*h.*	*m.*	*sec.*
	5	47	2	15	22	32
	2	35	6	20	34	21
diff.						
less.						
gr.						

ASTRONOMICAL TIME.

	deg.	min.	″	‴	⁗
	210	32	14	21	36
	321	37	3	14	27
diff.					
less.					
gr.					

MULTIPLICATION

Of divers denominations, in general is performed by multiplying each denomination by the given multiplier, and managing each product as you would a sum of the same amount in addition, placing the surplus of each under it, and carrying so many of the next denomination as was there found to be over; then the several products as placed under each of its kind, will be the product required, as for example:

(1)	£.	s.	d.
Multiply	5	14	8½
by			5
	£28	13	6½

The multiplier 5 being placed under the multiplicand £5..14..8½. proceed thus, 5 halfpence is 2½d. put down the ½d. and carry the 2 pence to 8 pence multiplied by 5 = 40, and 2 carried is 2d. this being equal to 3s. 6d. put down the 6d. in its place, and carry the 3s. to the 4s: (of 14) multiplied by 5 = 20, and 3 carried is 23s. put down the 3 in the units of shillings, and carry the 2 tens to the 1 ten of 14 multiplied by 5 = 5 tens, and 2 carried make 7 tens, = £3..10s. then place the odd 10s. with 1 by the 3, thus 13 under the shillings, carrying the £3. to the £5. × by 5 = 25 + 3 or £28. which 28 being placed under the pounds, the product will be £28..13..6½.

(2) Multiply £1..14..7¾ by 9.

£.	s.	d.
1	14	7¾
		9
£15	11	9¾

In this example the farthings, pence, and shillings are multiplied successively by 9; the products, as they are found, are respectively divided by 4, 12, 20, (or the tens of the shillings by 2;) the several remainders are written down, and the quotients carried. The pounds are multiplied as in Simple Multiplication; and the product is found to be £15..11..9¾.

EXAMPLES.

	£.	s.	d.			£.	s.	d.	
(3)	3	14	9½	× 2.	(9)	2	18	9	× 8.
(4)	1	17	8¾	× 3.	(10)	6	13	4	× 9.
(5)		18	11½	× 4.	(11)		15	10¼	× 10.
(6)	1	10	4	× 5.	(12)	9	7	8	× 11.
(7)	3	13	10¼	× 6.	(13)	3	19	7½	× 12.
(8)	1	2	9	× 7.	(14)	1	16	7	× 12.

(15) Multiply 2 *lb.* 9 *oz.* 15 *dwts.* by 3, 5, 8, and 10.
(16) Multiply 4 *yds.* 3 *qrs.* 2 *n.* by 4, 6, 9.
(17) Multiply 2 *a.* 3 *r.* 16 *pr.* by 4, 6, 9.

Though the above is a general rule, where your multiplier is a single figure, as it will frequently increase to two, three, &c. figures, this produces different varieties, as

When the multiplier consists of two figures, and a composite number, hence, seen in the multiplication table.

Rule. Multiply the multiplicand first by one component part as before taught, and this product again by the other, the last will be that which was required.

(18) Multiply £6..15..9¼. by 24.

Here 24 is compounded of 2 and 12; of 3 and 8; and 6 and 4, then with either of these as 2 and 12.

	£.	s.	d.
multiply thus	6	15	9¼
the multiplier is			24
the product by 2 =	13	11	6½
			12
the product by 24 =	162	18	6

Here follows the above example proved every way it can be done, as

	£.	s.	d.
First by 3 and 8.	6	15	9¼
			24
the product by 3.	20	7	3¾
			8
	£162	18	6

	£.	s.	d.
Again by 4 and 6.	6	15	9¼
			24
the product by 4.	27	3	1
			6
	£162	18	6

	£.	s.	d.	
Again by inverting any of the former as by 6 and 4.	6	15	9¼	
			24	
	40	14	7½	the product mul. by 4.
			4	
	£162	18	6	

When the multiplier is a prime number not found in the table.

Rule. Choose the nearest composite number in the table to your multiplier more or less, and multiply by it per last or variety II. noting the difference in your mind, and should the number chosen be more than your multiplier, *subtract* the product of the multiplicand by the difference, and if less, *add* the product of the multiplicand by the difference, *from*, or *to*, the product of the composite number chosen, and the *difference* in the former, or the *sum* in the latter will be the product required.

EXAMPLE.

	£.	s.	d.	
(19) Multiply	4	13	7½	by 38.
			38	
	18	14	6	
			9	
	168	10	6	
	9	7	3	
	£177	17	9	

Here I find from the table, 36 or 40 are equally near to 38.—36 being 2 less, and 40 being 2 more, then taking 36 I multiply by 4 and by 9, and find the product per variety II. to be £168..10..6. this wanting 2 to make 38, I multiply the multiplicand, £4..13..7½. by 2, and add the product £9..7..3. to £168..10..6. which gives the product of £4..13..7½ by 38 to be £177..17..9. as was required.

Again, taking the composite number 40,

being 2 more, thus,	£4	13	7½
I multiply 5 and 8			40
The product by 5 is =	23	8	1½
			8
The product by 40 is =	187	5	0
From this take £4..13..7½. × 2 =	9	7	3
Gives the product by 38 as before	£177	17	9

MORE EXAMPLES.

(20) Multiply £1..16..9½. by 34, 43, 58, 76, 87, 98, and 104, proving each.

(21) Multiply 2 *t.* 15 *cwt.* 3 *qrs.* 9 *lb.* by the same multipliers, and prove each.

(22) Multiply 3 *lb.* 11 *oz.* 14 *dwt.* 2 *gr.* by the same multipliers, and prove each.

(23) Multiply 3 acres, 2 roods and 24 perches, by the same multipliers, and prove each.

(24) Multiply 2 *yds.* 3 *qrs.* 3 *nails*, by the same multipliers, and prove each.

When your multiplier is composed of three figures, consisting of many hundreds, as for example, by 735, &c.

RULE. Multiply the given multiplicand first by 100, that is, by (10) and by (10,) by variety II., this product again by (7,) which, in the above instance, expresses the number of hundreds; again multiply the first product of 10 by (3), the next figure given, expressing the number of tens, and place this product under that by (700;) lastly, multiply the given multiplicand by (5,) expressing the number of units, and place this product under the other two; these three products added will be the product required.

EXAMPLE ILLUSTRATED.

(25)

	£.	s.	d.	
Multiply	1	17	4¼	the multiplicand to be multiplied.
by		735		as mentioned above.
	18	13	6½	=the product of £1..17..4¼. by 10.
			10	
	186	15	5	=pro. by 100 or of 10 by 10 again.
add	1307	7	11	=pro. by 700 or of 100 by 7.
	56	0	7½	=pro. by 30 or of 10 by 3.
	9	6	9¼	=pro. by 5 or multiplicand by 5.
	1372	15	3¾	=pro. by 735 as was required.

MORE EXAMPLES.

(26) Multiply £3..14..10½. by 482.
(27) Multiply £17..9..3¼. by 563.

QUESTIONS TO EXERCISE IN MULTIPLICATION.

(1) What will 2 yards of cloth cost at 6*s*. 10*d*. per yard?

(2) What will 5 yards of broad cloth cost at 18*s*. 6*d*. per yard?

(3) What will 6 ells of holland cost at 7*s*. 2½*d*. per ell?

(4) What cost 4 pounds of tea at 9*s*. 7*d*. per pound?

(5) What cost 3 yards of cambric at 10*s*. 9*d*. per yd.?

(6) What cost 8 pounds of sugar at 10¾*d*. per lb.?

(7) What cost 19 lb. of butter at 11½*d*. per lb.?

(8) What cost 23 cwt. of lead at 17*s*. 2¼*d*. per cwt.?

(9) What must be paid for 25 dozen of flour at 1*s*. 10¾*d*. per dozen?

(10) What is the price of 26 barrels of beer at 14*s*. 2*d*. per barrel?

(11) What cost 28 trusses of hay at 2*s*. 6½*d*. per truss?

(12) What will 30 weeks' wages amount to at 17*s*. 9½*d*. per week?

13) What will 35 pounds of gold amount to at 6..14..6. per pound?

14) What is the weight of 36 guineas, each 5 dwt. rains?

(15) What is the value of 42 pieces, each £3..12.?

(16) How many pounds sterling in 44 half-guineas?

(17) At 4*s.* 4*d.* per dollar, what is the value of 39 dollars?

(18) What quantity of gold is requisite to make 48 snuff-boxes, each to weigh 12 dwt. 14 grs.?

(19) What is the value of 44 bits, each 7½*d.*?

(20) What is the weight of 40 moidores, each 6 dwt. 22 grs.?

(21) What will 46 ounces of silver amount to, at 6*s.* 5½*d.* per ouuce?

(22) What cost 47 pieces of ivory at 13*s.* 3½*d.* each?

(23) What will 49¼ yards of shalloon cost at 1*s.* 11*d.* per yard?

(24) What will 52 bibles come to at 3*s.* 7*d.* each?

(25) At 30*s.* 2*d.* per ton, what is the price of 50½ tons?

(26) What cost 53 dozen of port wine at £1 .. 5 .. 9. per dozen?

(27) What will 58½ pounds of pins come to at 19½*d.* per pound?

(28) What is the price of 56¾ yards of serge at 3*s.* 3*d.* per yard?

A MERCER'S BILL.

(29)	*s.*	*d.*		£.	*s.*	*d.*
4 yards of silk ....at	13	3	per yard.			
7 yards of satin ....at	10	11	———			
3 yards of brocade ......at	17	0	———			
12 yards of lutestring ...at	6	1	———			
5 yards of velvet.........at	18	7	———			
11 yards of flowered silk at	15	6	———			
				25	16	10

DIVISION OF DIVERS DENOMINATIONS

Is performed by dividing each denomination by the divisor, beginning with the highest, and if any remainder, multiply it by that number which one of the higher contains of the next lower denomination, and proceed in like manner with each denomination; the quotients when placed under each denomination will be the quotient required. This rule supposes first the divisor to be a single figure.

EXAMPLE.

Divide £26..13..7½. by 6. Place them as in the margin, and dividing 6 into £26., the quotient is 4 times, and £2. over; place the quotient 4, now £4., under the £26., then multiply the 2 left by 20, the shillings in £1., the product is 40; to this add the 13 shillings, it makes 53 shillings, this divided by 6, the quotient is 8 times, and leaves 5*s.*; put the quotient 8 under the 13*s.*, and multiply the remaining 5*s.* by 12, the pence in 1 shilling, the product is 60 pence, to this add the 7 pence, the sum is 67 pence, which divided by 6, the quotient is 11, and leaves 1 penny; put the 11 pence under the 7, and multiply the 1 penny remaining by 4, the farthings in 1 penny, the product is 4, to which add the 2 farthings in the ½*d.*, the sum is 6 farthings; this divided by 6, the quotient is ¼*d.*, which being put under the ½*d.*, the work is done, and the quotient required is £4..8..11¼. To prove this, multiply the quotient by the divisor 6, and as the product is equal £26..13..7½., the quotient is right.

(1)

	£.	*s.*	*d.*
6)	26	13	7½
	4	8	11¼
For proof × by 6			
	26	13	7½

EXAMPLES.

(2) Divide £2..13..6. by 2.
(3) Divide £1..17..9. by 3.
(4) Divide £3..18. by 4.
(5) Divide £54..17..6. by 5.
(6) Divide £236..10. by 6.
(7) Divide £1..19..2¾. by 7.
(8) Divide £27..18..6. by 8.
(9) Divide £32..14. by 9.
(10) Divide 10 *cwt.* 3 *qrs.* 16 *lbs.* by 10.
(11) Divide 17 *yds.* 0 *qrs.* 3 *nails*, by 11.
(12) Divide £182..16. by 12.

To divide by a number which is greater than 12, but is the product of two or more factors, each less than 13.

RULE 1. Divide the given number, by Short Division, by one of the factors. 2. Divide the quotient by another factor. 3. Divide the result thus obtained by another factor, if there be so many: and thus proceed, whatever may be their number.

EXAMPLE.

Divide £59..13..3½. by 66.

In this example the factors are 6 and 11. In the division by 6, the quotient is £9..18..10½., and the remainder 2 farthings; and in the division of this quotient by 11, the quotient resulting is 18*s*. 0¾*d*., and the remainder 9: This remainder being multiplied by 6, the first divisor, and the product increased by the former remainder, 2, the true remainder is found to be 56 farthings, or 1*s*. 2*d*.—In the proof by Multiplication, this remainder must be added to the final product.

(13)

	£.	*s.*	*d.*	
6)	59	13	3½	
11)	9	18	10½	...2.
	0	18	0¾	...56 far.
			6	[or 1*s*. 2*d*.
	5	8	4½	
			11	
	59	12	1½	} add.
	0	1	2	
	59	13	3½	*Proof.*

ANOTHER EXAMPLE.

(14) Divide £59..16..8½. by 36.

Here 36 being compounded of 6 × 6, of 3 × 12, and of 4 × 9, hence having a choice of either, as for instance, 3 and 12, placing the divisor 36 on the left, proceed thus:

£. *s.* *d.*
3)59 16 8½ divide first by 3, and place the quotient under, then
by 12)19 18 10¾ = quotient, and 1 remains (viz. one farthing.)
Answer 1 13 2¾ = quotient required, and 7 remains, and the true remainder is 7 × 3 + 1 = 22 farthings = 5½*d*.

To prove this, multiply £1..13..2¾. by 36, and add in 5½*d*. = the remainder.

(15) Let £102..14..6. be given to be divided by 21.

£. *s.* *d.*
7)102 14 6
3)14 13 6 quotient by 7; divide again by 3.
4 17 10 *Ans.*

QUESTIONS TO EXERCISE.

(16) Divide £48..15..7½. by 24, 25, 48, 56, 63, 72, 84 and 132.

(17) Divide 48 *cwt.* 2 *qrs.* 21 *lb.* by the same divisors.

(18) Divide 52 *oz.* 9 *dwt.* 21 *grs.* by the same.

(19) Divide 121 acres, 2 roods, 19 perches, by the same, and prove each example.

When your divisor consists of a number not found in the Multiplication Table.

Rule. You must divide the different denominations by it, at once, and place the several quotients on the right of the dividend as in common division of whole numbers. See the following example:

(20) Let £347..15..10½. be divided by 23. Place them as follows, and first divide the pounds by 23, then divide in like manner any remainder with the odd shillings, lastly, the pence with the odd pence as follows.

Divisor. Dividend. Quotient.

23)£347..15..10½)£15..2..5.

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this 2 = the rem. from the pounds, after dividing them by 23, you must

multiply by 20 = the shillings in one pound, and add in the 15 shillings.

the product is 55 = dividend for shillings, which divided by 23 gives 2, which place to the quotient.

and leaves 9 shillings remaining; this 9

multiply by 12 = the pence in one shilling, and add in the 10 pence,

the product is 118 pence = the dividend for pence, and being divided by 23 gives 5, which put in the quotient,

and leaves 3 pence remaining; this 3

multiply by 4 = the farthings in 1*d*., and add in the ½*d*. = 2 farthings,

the product is 14, which being less than 23, it is the last remainder, and the quotient required is £15..2..5., and 14 farthings remain. Lastly,

Should your divisor consist of three, four, or any number of figures.

Proceed exactly as in the above example, and the proof in all cases will be, by multiplying the quotient by the divisor and adding in the remainder, as under.

	£.	*s.*	*d.*	
the quotient =	15	2	5	found as above, with 14 rem.
multiply by			23	the divisor; here, multiply by 24, and subtract 1.
	60	9	8	= the product by 4.
	362	18	0	= the product of 4 by 6, gives
	15	2	5	the product by 24, from which take £15..2..5. = the multiplicand once,
it leaves	347	15	7	= the product by 23, but
to this add			3½	= the 14 farthings remaining, and it
gives	347	15	10½	= the given dividend, as before.

(21) Divide £2074..6..9¼. by 597.

£.	s.	d.	£.	s.	d.	
597)2074	6	9¼(	3	9	5¾	*Ans.*
1791					6	
283			20	16	10½	
20					10	
5666			208	8	9	
5373					10	
293			2084	7	6	
12			10	8	5¼	
3525			2073	19	0¾	
2985				7	8½	
540			2074	6	9¼	*Proof.*
4						
2161						
1791						
370 farthings, or 7*s.* 8½*d.*						

Here, the first part of the quotient is £3., and the remainder £283..6., or 5666 shillings, from which, as in Simple Division, we obtain for quotient 9 shillings, and a remainder of 293 shillings and 9 pence, or 3525 pence. Dividing this by the divisor, we obtain for quotient 5 pence, and a remainder of 540¼ pence, or 2161 farthings, which gives a quotient of 3 farthings, and a remainder of 370 farthings, or 7*s.* 8½*d.* The work is proved by multiplying the quotient by 600 (6 × 10 × 10,) and subtracting three times the quotient, to obtain the product by 597, and finally, by adding the remainder, 7*s.* 8½*d.*, to the result.

MORE EXAMPLES.

(22) Divide £2461..13..7¼. by 29, 39, 47, 58, 67, 79, 143, 476, 740, 846, and 947, and prove them.

If the divisor consists of ¼, ½, or ¾.

Rule. Multiply the whole number of the divisor by 4, adding to the product 1 for ¼, 2 for ½, 3 for ¾, and it will give the divisor; which divide by as before, and the quotient multiplied by 4, will give the answer.

EXAMPLES.

(23) Suppose I give for 6¼ yards of cambric £4..10..7½., at what rate do I buy it per yard?

yds.		*£.*	*s.*	*d.*
6¼				
4	5)	4	10	7½
25	5)	0	18	1½
		0	3	7½
				4
Ans.		0	14	6

Here I reduce the divisor to quarters, namely 25. I divide £4..10..7½. by 5, and that result again by 5; then to balance, I *multiply* the result by 4, because I multiplied the divisor by 4.

Or, multiply *both* the dividend and divisor by the fraction, and divide as in the following case:

(24) Divide £64..1..4. by 7¾.

Divisor.		*£.*	*s.*	*d.*	
7¾		64	1	4	
4				4	
31	*New divisor.*	256	5	4	*New dividend.*

Use Long Division, as explained page 42.

£.	*s.*	*d.*	*£.*	*s.*	*d.*
31)256..	5..	4. (	8..	5..	4.
·8					
20					
165					
10					
12					
124					
...					

Having multiplied both divisor and dividend by 4, they are increased in equal proportion. I find then that I have to divide £256..5..4. by 31. Having placed them as in the example, I find that 31 is contained in 256 eight times, which I place as my quotient for pounds, the 8 that remain I multiply by 20, and take in the 5*s*., making 165, which I also divide by 31, and find the quotient to be 5*s*., and the remainder 10*s*.; they being shillings, I multiply by 12, (the pence in a shilling,) and take in the 4*d*., making 124 pence. I then see that 31 is contained in 124 four times, which I place to the quotient for pence. Had there been a remainder after finding the last quotient (4*d.*,) I should have drawn a line and placed the divisor under it, thus, suppose the remainder had been 5, I should have placed it $\frac{5}{31}$.

(25) As another example in this kind of division, let it be required to divide £30..2..10½. by 22¾.

$$\begin{array}{rl}
 & \text{£. } s. \ d. \\
22\tfrac{3}{4}) & 30..\ 2..10\tfrac{1}{2} \\
\underline{4} & \underline{\qquad\quad 4} \ \text{£. } s. \ d. \\
91) & 120..11..\ 6(\ 1..\ 6..\ 6 \\
 & \underline{91} \\
 & 29 \\
 & \underline{20} \\
 & 591 \\
 & \underline{546} \\
 & 45 \\
 & \underline{12} \\
 & 546 \\
 & \underline{546} \\
 & \ldots
\end{array}$$

Here a difference is made in the modes of Long Division that the learner may choose. With that exception, the mode is similar.

Here we multiply 22 by 4, the lower term of the fraction, and to the product 88, we add 3, the upper term.—We multiply the given dividend also by 4. Then, dividing this product by 91, we find £1 .. 6 .. 6. for the required quotient.—It may be observed, that these methods are equally applicable in Simple and Compound Multiplication and Division.

QUESTIONS,

SHOWING THE USE OF DIVISION.

(1) If a piece of broad cloth containing 24 yards cost £18. 6*s.* what did it cost per yard? *Ans.* 15*s.* 3*d.*

(2) Bought 39 *cwt.* of butter for £49. 14*s.* 6*d.* what did it cost per *cwt.*? *Ans.* £1. 5*s.* 6*d.*

(3) If a hogshead of wine cost £16. 16*s.* what is it per gallon? *Ans.* 5*s.* 4*d.*

(4) If 27 *cwt.* of sugar cost £47. 13*s.* 9*d.* what cost 1 *cwt.* *Ans.* £1. 15*s.* $3\frac{8}{9}$*d.*

(5) If 1 *cwt.* of sugar cost £3. 10*s.* what is it per *lb.*? *Ans.* 7½*d.*

(6) If I hold 120 acres of land, and my yearly rent be £87. what do I pay an acre? *Ans.* 14*s.* 6*d.*

(7) If 35 persons pay a reckoning of £6. 14*s.* 2*d.* what is the share of each? *Ans.* 3*s.* 10*d.*

(8) A prize of 1000 pounds is to be divided among 150 sailors, what is each man's share after deducting ⅙th part for the officers?

(9) If 125 ingots of silver, each of an equal weight, weigh 1347 *oz.* 11 *dwts.* 14 *grs.* what is the weight of 1 ingot?

(10) If 475 *cwt.* 1 *qr.* 14 *lbs.* be the weight of 27 *hhds.* of tobacco, what is the weight of 1?

(11) Bought 6½ pieces of tapestry, containing 237 ells Flem. 2 *qrs.* 2 *na.*, what is the length of 1 piece?

(12) What is the fourth part of 9 *yds.* 2 *ft.* 2 *in.*?

(13) Bought 4½ ounces of silver for £1..8..8¼.; what is the price of 1 ounce?

(14) Gave £3..17..7¼. for 6¼ yards of muslin; what is the price per yard?

(15) If 42 bottles of wine be sold for £4..16..3; what is it per bottle?

(16) A pound weight of standard gold bullion being coined into $46\frac{29}{40}$ sovereigns; required the full weight of one sovereign.

(17) Bought 39¼ yards of cloth for £29..15..3½., what did it cost per yard?

(18) What is serge per yard, if 43 yards cost £7..7..9¾.?

(19) If 1 cwt. of hops cost £4..15..8; how much are they per pound?

(20) Bought 8 cwt. of currants for £30..16..0.; what did they cost per pound?

(21) Bought 50 chests of oranges, each containing 1000, for £260..8..4.? what was the price of the oranges, one with another?

(22) What is the value of 1 *lb.* of tea, if 63 chests, each weighing 93 *lbs.* neat, cost £3106..9..9¾.?

(23) If half a dozen tea-spoons weigh 4 *oz.* 18 *dwts.* 18 *grs.*; what does each spoon weigh?

(24) If 42 loaves of sugar, of equal weight, weigh 8 *cwt.* 3 *qrs.* 7 *lbs.*; what is the weight of one loaf?

INSTRUCTIONS.

QUEST. 1. Divide the cost by 6, and the result by 4.

QUEST. 2. Divide by 39, use long division.

QUEST. 3. Remember there are 63 gallons in a hogshead.

QUEST. 4. You know what 2 numbers multiplied together will produce 27.

*QUEST. 5. A *cwt.* contains 112 *lbs.*, use long division.

QUEST. 6. Divide the yearly rent by such numbers, as when multiplied, would make 120.

QUEST. 7. In dividing by 35, use long division.

QUEST. 8. Divide £1000. by 6, and the remainder divide again by 150, the quotient will be the answer.

QUEST. 9. Having divided 1347 by 125, multiply the remainder by the number of *penny weights* in a *lb.*, observing to take in the 11 *dwts.* divide again, and multiply the remainder by the number of grains, contained in a *dwt.*

QUEST. 10. Act in a similar manner with this question, observing that your question is *cwt. qrs.* and *lbs.*, instead of *ounces*, *penny-weights*, and *grains*.

QUEST. 11. Reduce both the pieces, ells, quarters, and nails to halves, and then divide one by the other. Your divisor will be 13, it may be done in one line easily.

QUEST. 13. Divide twice the money by twice the number of ounces, and the quotient will be the price of one ounce. See Division of Integers, by a mixed number.

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* I do not give in this work, (it being intended *to teach the simple rules only*), any abbreviated modes. This question, by the most simple rules as taught in Dr. Willcolkes's and Messrs. Fryers' System, would be to multiply 9*s.* 4*d.* (equal to 112 *lbs.* at 1*d.*) by 7½*d.* thus,

s. *d.*
9 4 at 1*d.*
7½
£3 10 0

Quest. 14. Divide 4 times the money by 4 times the given number of yards, and the quotient will be the price of one yard.

Quest. 16. If $46\frac{29}{40}$ sovereigns weigh 1 *lb.*; then, multiplying each of these by 40, the results will be 1869 sovereigns, weighing 40 *lbs.* Troy. Therefore, to find the weight of a sovereign, when new from the mint, divide 40 *lbs.* by 1869, and the quotient will be 5 *dwts.* $3\frac{515}{1869}$ *grs.* being the full weight of a sovereign.

REDUCTION.

Reduction being the changing or reducing numbers or quantities of one denomination to that of another, it is performed by *multiplication* and *division*, thus, when performed by multiplication, it is called *Reduction descending*, that is, the bringing numbers from a higher denomination to a lower, and when by *division*, it is called *Reduction ascending*, that is, the bringing numbers from a lower to a higher denomination, but in each instance still reserving their full value, and

To bring numbers from a higher denomination to a lower, the Rule is,

Multiply the higher denomination by that number which one of the higher contains of the next lower, observing to add to this product any odd number of the same name contained in the given quantity to be reduced, and in that manner proceeding from one denomination to another until the last, the last product will be that which was required. This is *Reduction descending*, but

To bring numbers back again from a lower denomination to a higher, the Rule, in this case, will be to

Divide the given denomination to be reduced or changed by that number which one of the greater contains of the next less, and in that manner proceeding through each denomination to the highest, the last quotient with the several remainders will be the denomination required; and after division, if any thing remains, it is of the same name with the last dividend. *And this is Reduction ascending.*

METHODS OF PROOF.

1. To the answer found by either of the preceding rules, apply the other rule, and if the result be the same as the given number, the work is correct.

2. The operation may be proved by the methods of proving Multiplication or Division.

The reasons of the rules will be explained in the illustrations of the

EXAMPLES.

(1) In £36. how many shillings?
Here 36×20=720 shillings.

In 720 shillings how many pounds?
Here 720 divided by 20, quotes £36.
and proves the first question.

£36

20

720 = shil.

(2) In £49. 18*s*. how many shillings and pence?
Here £49. multiplied by 20, adding in 18 shillings, gives 998 shillings, and 998 shillings multiplied by 12, gives 11976 pence.

(3) Reduce £59. to farthings.
In this example the pounds are multiplied by 20, to reduce them to shillings, because there are 20 shillings in each pound. The shillings, in like manner, are multiplied by 12, to reduce them to pence, and the pence by 4, to reduce them to farthings, because there are 12 pence in each shilling, and 4 farthings in each penny. Hence, it appears, that in £59. there are 1180 shillings, 14160 pence, or 56640 farthings.

£.

59

20

1180 shillings.

12

14160 pence.

4

4)56640 farthings.

12)14160 pence.

2|0) 118|0 shillings.

£59 proof.

The operation is proved by dividing successively by 4, 12, and 20, the former multipliers, in a reversed order; and the work is correct, since the final quotient, £59., is the same as the quantity given in the question.

(4) Reduce £94..12..8¼. to farthings.

£. s. d.
94 12 8¼
20
1892 shillings.
12
22712 pence.
4
90849 farthings.

4)90849
12)22712¼
2|0)189|2 8¼
£94 12 8¼ proof.

In this example in the multiplication by 20, 12 shillings are added to the product; in the multiplication by 12, 8 pence are added, and in the multiplication by 4, 1 farthing is taken in. Hence, the answer is 90849 farthings.

(5) Reduce 83918 farthings to pounds, &c.

In this example, the farthings are divided by 4, because in the same sum there are 4 times as many farthings as there are pence: for a similar reason, the pence are divided by 12, to reduce them to shillings, and the shillings thus found by 20, to reduce them to pounds. Hence, it appears that 83918 farthings are equivalent to 20979½ pence, 1748 shillings and 3½*d.*, and to £87..8..3½.

4)83918 farthings.
12)20979½
2|0) 174|8 3½
£87 8 3½ *Ans.*

(6) In 19 pounds, how many shillings, pence, and farthings?

(7) In 55 guineas, how many shillings, pence, and farthings?

(8) Reduce £54..11..9¼. into farthings.

(9) Reduce £77..11..10½. into halfpence.

(10) Reduce £94..14..8. into pence.

(11) Reduce £47..14..4. into two-pences.

(12) In £34..11..9. how many three-pences and pence?

(13) In £47..19..8. how many groats, pence, and farthings?

(14) In £108..11..6. how many sixpences?

(15) How many crowns, half-crowns, shillings, sixpences, and pence, are in £54.?

(16) Reduce £74..13..9. into shillings, three-pences, and farthings.

(17) In 11520 farthings how many pence, shillings, and pounds?

(18) In 17880 pence how many pounds?

(19) Reduce 100800 farthings into guineas

(20) In 50400 halfpence how many pounds?

(21) In 12050 shillings how many crowns and pounds?

(22) Reduce 311040 pence into groats, shillings, crowns, and pounds.

(23) In £102..16..3. how many pieces of coin, each 7*s.* 3½*d.* in value?

(24) In 7494 dollars, at 4*s.* 6*d.* each, how many groats, shillings, half-crowns, crowns, and pounds?

TO ASSIST THE LEARNER.

QUEST. 6. Multiply 19 by 20, the result by 12, and that result by 4.

QUEST. 7. Each guinea consider as 21 shillings, and act as in example, question 6.

QUEST. 8. See example, pages 73 and 74.

QUEST. 9. Having reduced the £77..11..10. to pence, multiply the product by 2, and take in the halfpenny.

QUEST. 12. Having reduced £34..11..9. to pence, you have found the number of pence, to find the number of three-pences, divide by 3.

QUEST. 13. *Divide* the pence contained in £47..19..8. to find the groats, (a groat being 4*d.*) and multiply the before-mentioned pence by 4, to find the farthings.

QUEST. 14. Multiply the shillings in £108..11*s.* by 2, and take in *one* for the 6*d.*

QUEST. 15. Multiply £54. by 4, to find the crowns that result by 2, to find the half-crowns that by 2½, to find the shillings that by 2, to find the sixpences, and that by 6 to find the pence.

QUEST. 17. Divide by 4, 12, and 20.

QUEST. 18. Divide by 12 and 20.

QUEST. 19. Divide by 4, 12, and *twenty-one*.

QUEST. 23. Reduce £102..16..3. to halfpence, and also 7*s* 3½*d.* to halfpence, divide the greater by the lesser.

If the foregoing questions should not be found sufficient, the following, or part of them may be solved.

(25) In 12051 groats, 8034 sixpences, and 4017 shillings, how many pounds? find separate answers for each.

(26) In 21 guineas, how many pence?

(27) In 5733 pence, how many guineas?

(28) In 1264 groats, how many three-pences, and prove it?

(29) There is a debt due of £47..8..9. which is to be paid in dollars of 4*s.* 6*d.* each; how many dollars will it require to discharge it?

(30) How many pieces of 8*s.* 6*d.* each will pay £143..8..10., and what is the remainder?

(31) Find the number of times 3*s.* 1½*d.* is contained in £49..7..6.

(32) Having received £180. to distribute amongst 345 poor persons, how much can I give to each?

REDUCTION OF TROY WEIGHT.

(33) Reduce 1 *lb.* 5 *oz.* 12 *dwts.* 13 *grs.* to grains.

```
lb. oz. dwts. grs.
 1   5   12   13
12                     24)8461
--                     -------
17                     2|0)35|2   13
20                     --------
---                      12)17   12   13
352                      ------
 24                      lb. 1    5   12   13  Proof.
----                     ------
1421
704
----
8461 grains. Ans.
----
```

In this example the multipliers are 12, 20, and 24, because in each pound there are 12 ounces, in each ounce 20 pennyweights, in each pennyweight 24 grains; and the 5 ounces, 12 pennyweights, and 13 grains, are taken in, as in the former examples.

EXERCISES.

(34) In 354 *lb.* how many grains?

(35) In 7 *lbs.* 2 *oz.* 15 *dwts.* how many pennyweights?

(36) In 482 *lbs.* 7 *oz.* 13 *dwts.* how many grains?
(37) In 2039040 grains of silver, how many *lbs.*?
(38) In 1735 *dwts.* how many *lbs.*?
(39) In 2779992 grains, how many *lbs.*?

REDUCTION OF AVOIRDUPOIS WEIGHT.

(40) Reduce 27 *cwt.* 2 *qrs.* 22 *lbs.* to pounds.*

cwt. qrs. lbs.	
27 2 22	4)3102
4	7) 775 2
110	4) 110 22
28	*cwt.* 27 2 22 *Proof.*
902	
220	
3102 pounds. *Ans.*	

In this example, the hundreds being multiplied by 4, the product is quarters, because there are 4 quarters in each hundred; and the quarters being multiplied by 28, the product is pounds, because in each quarter there are 28 pounds. The odd quarters and pounds are taken in, as the operation proceeds.

EXERCISES.

(41) In 124 *cwt.* 1 *qr.* 13 *lb.*, how many pounds?
Ans. 13929.
(42) In 275 *cwt.* 3 *qrs.* 27 *lb.*, how many pounds?
Ans. 30911.
(43) In 75 *cwt.* 2 *qrs.* 27 *lb.*, how many pounds?
Ans. 8483.
(44) In 72 *cwt.* 1 *qr.* 8 *lb.*, how many pounds?
Ans. 8100.

CLOTH MEASURE.

(45) In 76 *yds.* 1 *qr.* 3 *n.*, how many nails?
(46) In 1207 nails, how many yards?

---

* The Author is acquainted with several abbreviated methods of reducing *cwts.*, &c., to *lbs.*, and also with other short modes, but as they are laid down in the clearest manner in *Willcolkes's and Fryers' united System*, they are not introduced here; this Work being suited more to afford practice than brevity.

(47) In 720 yards, how many ells English?
(48) In 576 ells English, how many yards?
(49) In 576 ells English, how many ells Flemish?
(50) In 960 ells Flemish, how many ells English?

OCCASIONAL ASSISTANCE.

QUEST. 45. Multiply the 76 yards by 4, and take in the 1 quarter, the product multiply by 4, and take in the 3 nails.

QUEST. 46. As 16 nails make 1 yard, divide 1207 by 16, and by 4, and the product also by 4.

QUEST. 48. As an English ell is 5 quarters, multiply 576 by 5, and divide the product by 4, or to 576 add one fourth of itself.

REDUCTION OF LAND MEASURE.

(51) Reduce 37 English acres, 3 roods, 12 perches, to square yards.

a. *r.* *p.*
37 3 12
4
151 roods.
40
6052 perches.
$30\frac{1}{4}$
181560
1513
183073 yards. *Ans.*

The acres are multiplied by 4, and the roods taken in; the roods are then multiplied by 40 and the perches taken in, and the perches are multiplied by $30\frac{1}{4}$, (the number of square yards in a perch), consequently the result is 183073 yards.

EXAMPLES.

(52) In 77 acres, 1 rood, 14 perches, how many perches? *Ans.* 12374.
(53) In 17280 perches, how many acres? *Ans.* 108.
(54) How many roods are there in 845 perches?
(55) Reduce 14 acres, 0 roods, 38 perches, to perches.

LIQUID MEASURE.

(56) In 45 hogsheads, how many gallons?

$$\begin{array}{r} 45 \\ \underline{63} \\ 135 \\ \underline{270} \\ \underline{2835} \end{array}$$

As a hogshead contains 63 gallons, multiply the hogsheads by that number of gallons.

(57) In 4080 pints, how many gallons?

(58) In 7 hogsheads and 25 gallons, how many gallons, quarts, and pints?

(59) I had 72 hogsheads and 24 gallons of wine, but having sold 18 hogsheads, 28 gallons, and 5 pints, I desire to know what quantity remains in hogsheads, gallons, and pints; and also in pints.

DRY MEASURE.

(60) In 22 quarters of corn, how many bushels?

$$\begin{array}{r} 22 \\ \underline{8} \\ \underline{176} \end{array}$$

Multiply the quarters by the number of bushels contained in 190.

(61) How many pecks are in 161 gallons?

(62) In 2906 pecks, how many bushels?

(63) In 367 quarters of corn, how many bushels?

(64) How many pecks are in 442 *qrs.* 5 *bu.* 3 *pks.* of wheat?

(65) Reduce 278 *ch.* 34 *bu.* of coals to bushels.

(66) In 3898 pecks of wheat, how many quarters?

(67) In 24967 bushels of oats, how many pecks?

(68) How many quarts are in 2 bushels?

OCCASIONAL ASSISTANCE.

QUEST. 63. Multiply the quarters by 8, and take in the bushels; multiply the bushels by 4, and take in the pecks.

QUEST. 65. Multiply the chaldrons by the bushels one contains, and take in the 34 bushels.

TIME.

(69) Reduce 14 hours to minutes.

14 hours.
60 minutes.

Ans. 840 minutes.

(70) Reduce 9 days to hours and minutes.

(71) Find the number of minutes in 14 weeks, 5 days, 7 hours, and 45 minutes.

(72) Reduce 2 years and 25 days to days; supposing the year to be 365 days.

(73) Find the number of years, months, (of 30 days each) weeks, days, and minutes in 1476850 seconds.

OCCASIONAL ASSISTANCE.

QUEST. 70. Multiply the hours in a day by 9 days, and that product by the minutes in an hour.

QUEST. 71. Multiply the 14 weeks by 7, and take in the days, that result by 24, and take in the hours, that result by 60, and take in the seconds.

QUEST. 72. Multiply the days in *one* year by 2, and take in 25.

GENERAL QUESTIONS TO EXERCISE.

(1) In 12 bags of wheat, each 2½ *cwt.*, how many stones, and allowing 20 stones to the barrel, how many barrels? *Ans.* 240 stones and 12 barrels.

(2) *In £18..3..4., how many crowns, half-crowns, shillings, 4 pences, and 3 pences, and of *each an equal* number? *Ans.* 40 of each.

(3) How many minutes since the commencement of the Christian era, supposing it to be the 31st of December, 1838? *Ans.* 966052800.

(4) How often will a wheel, 16 feet 8 inches in circumference, turn round in running from London to Greenwich, supposing the distance to be 7 miles?
Ans. 2217⅗ times, or 2217, and leave 120 feet.

(5) Suppose a merchant has orders to ship 120 *cwt.*

---

* Find the number of pence in a crown, a half-crown, a shilling, a four-pence, and a three-pence, and divide the *pence* contained therein into £18..3..4., the quotient will be the answer.

2 *qrs.* and 4 *lb.* of beef in barrels, each to contain 150 *lbs.*; how many will he require? *Ans.* 90 barrels.

(6) How many pounds in 54 hogsheads of sugar, supposing each to weigh 17½ *cwt.* *Ans.* 105840.

(7) A tobacconist has a hogshead of tobacco containing 17 *cwt.*, and wishes to divide it into parcels containing * 5 *lb.*, 8 *lb.*, 12 *lb.* 14 *lb.*, and 28 *lb.*, and to have of each an equal number; how many can he have? *Ans.* 28, and 28 *lbs.* over.

(8) Sold 5 packs of wool, viz.

		cwt.	*qrs.*	*lb.*
No. 1.	...	5	1	17
2.	...	5	3	14
3.	...	6	2	4
4.	...	4	3	25
5.	...	6	1	12

I want to know how many stones are in them, supposing each stone to be 16 *lbs.*? *Ans.* 204.

(9) How many boxes, each to hold 12 *lb.* may be filled out of a hogshead of tobacco containing 7½ *cwt.*? *Ans.* 70.

(10) Received from Jamaica 56 hogsheads of sugar, each 12 *cwt.* 1 *qr.* 10 *lb.* (100 *lb.* being their *cwt.*) how many *cwt.* here of 112 *lb.*? *Ans.* 617 *cwt.* 2 *qrs.*

THE RULE OF THREE.

THE Rule of Three Direct, teaches by three given numbers, to find a fourth, which shall have the same ratio, or proportion, to the second, as the third has to the first; that is, if the first be greater than the third, the second will be greater than the fourth; and if the first be less than the third, the second will be less than the fourth.

RULE. State the question by placing the numbers in such order that the first and third may be of one kind, and the second the same as the number required; then bring the first and third numbers into one name, (if they are not already so) and the second into the lowest denomination mentioned. Multiply the second and third numbers together, divide the product by the first, and the quotient will be the answer, in the same denomination as the second number.

---

* See instructions given for the working of Question 2, page 80.

If there be a remainder after division, it is always of the same name as the lowest denomination of the middle number, and must be reduced to the next inferior denomination, then divide again by the first number, till you come to the lowest denomination, which the middle number admits of. The several quotients taken together will be the answer. The method of proof is by changing the order of the stating. The Rule of Three is by some called the Rule of Proportion, and by others the Golden Rule, for its extraordinary use; both names may justly be considered perfectly applicable,

The first number is always known by the words, *if*, *at*, or *per*; the second is easily known by the word *cost*, (for it depends upon the first;) the third number is always known by these, (or such like) words, *How much? How many? What will? How long? How far?* &c. &c.

EXAMPLES.

(1) If three yards cost 9*s.*, what will six yards cost at the same rate, or proportion?

yds.		*s.*		*yds.*
3	cost	9	what cost	6
		6		
	3)	54		
		18*s.* *Ans.*		

Multiply the second and third terms, and divide their product by the first. The quotient is the fourth proportion sought, of the same name and kind as the second.

If the first number be 1, the answer or fourth, is found by multiplying the second and third, and if the second or third be 1, the fourth will be found by dividing the other number by the first.

(2) What cost 327 yards of canvass, at 8*d.* per yard?
Ans. £10..18..0.

yd. *d.* *yds.*
Thus: As 1 is to 8 so is 327 to the answer.

As the first number is 1, the second and third are multiplied; the product being *pence*, it is divided by 12 and 20, to reduce it to pounds, &c.

8
12)2616 pence.
2|0) 21|8 shillings.
£10..18..0 *Ans.*

(3) What will 17 tons of tallow cost, at £25. per ton?

NOTE. This question is to be solved in the same way as question 2, but the result will be pounds; therefore it is *not* to be divided. *Ans.* £425.

(4) At 7*s.* per *lb.* what will 128 *lb.* of tea cost?

lb. *s.* *lb.*
As 1 : 7 :: 128

Multiply 128 by 7, the result will be shillings, and as your first term is one, you have only to divide 896 shillings by 20, to prodnce the answer £44..16..0.

(5) What is the value of 139 barrels of barley, at 6*s.* per barrel? *Ans.* £41..14.

(6) How much is the price of 178 *lb.* of merchandize, at 17*d.* per *lb.*? *Ans.* £12..12..2.

NOTE. When the second number is of different denominations, bring it to the lowest mentioned, and the fourth will be found in *the same name to which the second is reduced,* which reduce back to the highest possible.

EXAMPLES.

(7) Let it be required to find what 178 yards of linen will cost, at 4*s.* 8*d.* per yard.

yd. *s.* *d.* *yds.*
As 1 : 4 8 :: 178
12 56
56 pence. 1068
890
12)9968 pence.
2|0)83|0..8 pence.
£41..10..8 *Ans.*

(8) What will 37 gallons of wine cost, at 9*s.* 10*d.* per gallon? *Ans.* £18..3..10.

(9) What will 24 yards of cloth cost, at 16*s.* 7*d.* per yard? *Ans.* £19..18..0.

(10) What is the amount of 175 pieces of coin, at 2*s.* 8½*d.* each?

pie.   *s.*   *d.*   *pieces.*
As 1 : 2 8½ :: 175
12   65
32   875
2   1050
half-pence 65   2)11375 half-pence.
12)5687½ pence.
2|0)47|3 11½
£23 13 11½ *Ans.*

Here the second term (2*s.* 8½*d.*) is reduced to the lowest denomination (halfpence), the 175 pieces being multiplied by 65, the 11375 produced are also halfpence, which according to Reduction, are divided by 2, 12, and 20.

The same method may be adopted in working the following:

(11) What is the value of 462 pieces of coin, supposing each to be worth 4*s.* 9½*d.*? *Ans.* £110..4..1½.

(12) I have purchased 143 *oz.* of silver, at 4*s.* 10¾*d.* per *oz.*, what is the amount? *Ans.* £35..0..1¼.

NOTE. When fractions occur in both quantity and price, as in the following, proceed thus:

(12) What must I pay for 75½ bushels of wheat, at 3*s.* 10½*d.* the bushel?

```
bush.    s. d.       bush.
As 1 is to 3 10½ so is 75½ to the answer.
  2      12            2
 ---    ----         ----
  2      46           151
          2            93
        ----         ----
         93           453
                     1359
                     ----
                  2)14043
                   ------
                   2)7021½
                    ------
                 12)3510¾ pence.
                  -------
                 2|0)29|2 6¾
                    ---------
                    £14 12 6¾ Ans.
                    ---------
```

EXPLANATION.

The third number, (75½ bushels), is reduced to the lowest denomination, namely, half bushels, the first term is reduced also to half bushels, and the middle term or number is reduced to the lowest denomination, (halfpence) the product of the second and third numbers 14043 is then divided by the first number, which produces 7021½ halfpence, *being of the same denomination as the second term was left in, namely, halfpence,* and to reduce the same to pounds, shillings, and pence, it is divided by 2, 12, and 20.

NOTE. Though it is necessary to explain the Rule of Three, that it may be found suited to the meanest capacity to introduce questions, such as the foregoing and following, yet on reference to Dr. Willcolkes's and Messrs. Fryers' System of Arithmetical and Mental Calculations, it will be seen that nine-tenths of the questions, commonly occurring in that Rule, may be solved in about one tenth of the figures necessary by the Rule of Three, as for example, the last question would be solved thus :

75½ at 3*s*. 10½*d*. each.

```
           £2  6  6     13s. 6¾d. to
                  6       take in.
          ---------
Ans. £14 12  6¾.
     -----------
```

See Rule, page 52, and examples on the following pages. *Sixth Edition.*

(14) If a vessel containing 7¼ gallons of ale cost 4*s.* 6*d.*, what should be paid at the same rate for 142¼ gallons? *Ans.* £4..8..8$\frac{15}{29}$.

DIRECTIONS.

Having reduced your three terms to their proper denominations, &c. you will find that there will be a remainder of 15, under which draw a line, and place your divisor, thus: $\left.\begin{array}{l}15 \text{ remainder} \\ \overline{29} \text{ divisor}\end{array}\right\}$ by which you have the exact fraction, and which is very near a halfpenny.

(15) If 8 yards, 3 quarters of cloth cost 31*s.* 8*d.*, what will 70 yards, 1 quarter cost? *Ans.* £12..14..2$\frac{30}{35}$. or £12..14..2¾.

(16) If 9 *cwt.* 2 *qrs.* of sugar, cost £26 .15..2. what must be paid at the same rate for 115 *cwt.* 3 *qrs.*?

Stated thus: If 9 *cwt.* 2 *qrs.* cost £26..15..2. what cost 115 *cwt.* 3 *qrs.* *Ans.* £326..0..7.

(17) If 115 *cwt.* 3 *qrs.* of hops cost £326..0. 7. what is the value of 9 *cwt.* 2 *qrs.*? *Ans.* £26..15..2.

(18) If 2 *cwt.* 3 *qrs.* 14 *lb.* of any thing cost £6..14..2., what will 12 *cwt.* 3 *qrs.* cost? *Ans.* £29..15.

(19) If 2 *cwt.* 3 *qrs.* 21 *lb.* of tallow cost £6..1..8., what will 35 *cwt.* 1 *qr.* come to? *Ans.* £73.

(20) If for £73. I can buy 35 *cwt.* 1 *qr.* of sugar, what should I pay for 2 *cwt.* 3 *qrs.* 21 *lb.* at the same rate? *Ans.* £6..1..8.

(21) If £6..14..2. be paid for 2 *cwt.* 3 *qrs.* 14 *lbs.* of cheese, what quantity should be given for £29..15..0.? *Ans.* 12 *cwt.* 3 *qrs.*

(22) I have paid £6..4..11¼. for the carriage of 17 *cwt.* 3 *qrs.* 11 *lb.* of merchandize, what is the carriage of a *cmt.* (of 112 *lb.*)? *Ans.* 7*s.*

(23) At 7 shillings the *cwt.* what should be paid for the carriage of 17 *cwt.* 3 *qrs.* 26 *lb.* *Ans.* £6..5..10½.

(24) Bought a quantity of tea, for £62..13..4., at 5*s.* 4*d.* per *lb.*, how many *lbs.* had I? *Ans.* 235 *lbs.*

NOTE. The answer to the foregoing will be found by dividing the *pence* in 5*s.* 4*d.*, into the pence contained in £62..13..4.

The following may be worked in a similar way.

(25) Bought a ton of iron, for £14. 10*s.*, what quantity may I have for £420.?

Ans. 28 *tons* 19 *cwt.* 1 *qr.* 6$\frac{22}{29}$ *lb.*?

```
         £.  s.     ton.      £.
Thus: As 14. 10. is to 1 so is 420. to the answer.
             20                20
          -----             ------
          s. 290         290)8400(28 tons.
                             2600
                             ----
                              280
                                20
                            ------
                         290)5600(19 cwt.
                              2700
                              ----
                              ··90
                                 4
                             -----
                          290)360(1 qr.
                              ----
                               ·70
                                28
                             -----
                          290)1960(6 lbs.
                              ----
                              22|0
                              ----
                              29|0 fraction.
                              ----
```

Having reduced the first and third numbers to one denomination, (shillings) there is no occasion to multiply the second and third terms together, as the second term or number is but 1, the object is to find how many times 290 shillings is contained in 8400 shillings; I find 28 times, therefore that 28 must be tons; the first remainder (280) being tons. I reduce to *cwts.* and divide again by 290; the quotient I find to be 19 *cwt.*, and that remainder (90) I reduce to quarters and divide again, the 70 left I reduce to *lbs.*, which having divided, I find there is a remainder of 220, under which I place the divisor (290) and obtain a fraction (by cutting off a cipher from both numerator and denominator) of $\frac{22}{29}$ or about $\frac{3}{4}$ of a *lb.* The work may be considerably abbreviated by cancelling a cipher from both divisor and dividend in the first instance. It will be seen that the short method of long division, see page 42, has been used.

(26) What quantity of wine may be purchased for £456. at £16..16. per hogshead?

Ans. 27 hogsheads, 9 gallons.

(27) A lady bought a silver tankard for ten guineas, at *5s. 5d.* per ounce, what is the weight of it?

Ans. 38 *oz.* 15 *dwts.* $9\frac{3}{13}$ *grs.*

(28) Bought a hogshead of tobacco for £17..9..0. which weighed 5 *cwt.* 2 *qrs.* 16 *lbs.*; find the price of 5 hogsheads, containing 23 *cwt.* 1 *qr.* 3 *lbs.*

Ans. £71..19..7½.

(29) If 17 *cwt.* 2 *qrs.* 16 *lbs.* of any thing cost £23..10..0., what does one *cwt.* stand me in?

Ans. £1..6..8. nearly.

(30) If for £24..16..8. I buy 18 *cwt.* 2 *qrs.* 16 *lbs.* of sugar, what must I pay for 1 ton? *Ans.* £26..12..9¾.

(31) If I sell 21 *cwt.* 1 *qr.* 21 *lbs.* of beef for £8.; what will 650 *cwt.* 2 *qrs.* 19 *lbs.* come to at that rate?

Ans. £242..18..4.

(32) If £8. will purchase 21 *cwt.* 1 *qr.* 21 *lbs.*, what will £242..18..4 pay for? *Ans.* 650 *cwt.* 2 *qrs.* 19 *lbs.*

By the pupils getting acquainted with the foregoing examples in Proportion, any of the questions ordinarily occurring may be solved, I shall however in the following pages introduce abbreviated modes of performing questions in the Rule of Three.

ABBREVIATED METHOD OF PERFORMING QUESTIONS IN THE RULE OF THREE,

BEING PAGES 74—77 OF DR. WILLCOLKES'S AND MESSRS. FRYERS' NOVEL SYSTEM OF ARITHMETICAL AND MENTAL CALCULATIONS.—*Sixth Edition.*

Rule.—Divide any of the Quantities or Terms by one of the other, and *multiply* or *divide* the other Quantities or Terms by it, as the nature of the case may require—which the following examples will clearly explain.

EXAMPLES.

Suppose I have bought 64 gallons of brandy for £127..10..6. what should I pay for 8 gallons more at the same rate?

Thus: 8 into 64 goes 8 times; therefore by taking the ⅛ of what I paid for 64 *gals*, I have the result.

8 of 64 is ⅛)£127 10 6 for 64 *gals.*
£15 18 9¾ *Ans.*

Suppose 6 *cwt.* of sugar cost £15..9..2. what must I pay for 48 *cwt.*?

Here the quantity required to be found is 8 times as much the quantity of which you know the amount; therefore, you have to multiply £15..9..2. by 8. *Ans.* £123..13..4.

Suppose 4 yards to have cost me 16*s.* 4*d.* what must I pay for 25½ yards?

In this case I commence as I did in the first question, by dividing 25½ by 4, which I find goes 6 times, and leaves 1½; which 1½ being ⅜ of 4, I may multiply 16*s.* 4*d.* by 6⅜, though on second consideration, as it makes no difference which term is divided by the other, I say that as 16*s.* 4*d.* is the price of 4 yards, if I divide 16*s.* 4*d.* by 4, I shall by that means ascertain the price of one yard to be 4*s.* 1*d.*, and by any of the previous methods find the value of 25½ yards at that price.

s. *d.*
4) 16 4 price of 4 yards.
4 1 price of 1 yard.

Then 25½ at 1*s.* = £1 5 6
4 1
£5 4 1½

2*s.* 1½*d.* to take in, being the amount of 25½ at 1*d.*

Or thus:

4*s.* 1*d.*=49*d.*; as shillings=£2 9
25½ is 2⅛ doz. 2⅛
£5 4 1½

6*s.* 1½*d.* take in, being the ⅛ of £2. 9*s.*

The three preceding Examples may answer to explain the methods, as they are each done in different ways.

If for 8 yards of cloth I have paid £3..4..6. what should I pay for 20 yards?

Thus: 8 into 20 goes 2½ times; therefore multiply £3..4..6. by 2½. *Ans.* £8..1..3.

If 14 men can build a wall in 153 days, how many men may perform a similar piece of workmanship in 42 days?

I here say 14 into 42 goes 3 times, and divide 153 by 3. *Ans.* 51 men.

I have purchased 120 acres of land for £4260. what, at the same rate, must I give for 16 acres?

In this case the answer must be in proportion to 4260 as 16 is to 120 *much less.* I divide 120 by 16, which I find to go 7½ times. I therefore multiply 4260 by 2, and divide it by 15. What is the result?

QUESTIONS FOR PRACTICE.

If 4 yards of cloth cost £4..9..2. what is the value of 24 yards? *Ans.* £26..15.

Supposing for 72 *cwt.* I pay £84..12..6. what must I pay for 9 *cwt.*? for 12 *cwt.*? for 8 *cwt.*? for 6 and for 10 *cwt.*?

For 9 *cwt.*	*Answers.*	£10	11	6¾
For 8 *cwt.*		9	8	0⅔ or ½
For 6 *cwt.*		7	1	0½
For 10 *cwt.*		11	15	0¾
For 12 *cwt.*		14	2	1

Questions to any extent in the Single and Double Rule of Three, may be worked in the same manner.

We insert one example in Double Rule of Three.

If £20. in 6 months gain £4..9..2½. what will £80. gain in 9 months?

£20	80
6	9
120 gain £4..9..2½.	720

By multiplying the 20 by 6, and 80 by 9, it reduces the operation to simply asking if 120 gain £4..9..2½. what will 720 gain? Merely divide 720 by 120, and multiply £4..9..2½. by that product. Thus:

12,0)72,0	£4 9 2½
6	6
	£26 15 3 *Ans.*

QUESTIONS TO PRACTISE THE RULES OF PROPORTION.

If 5 men in 6 days earn £7..4..2. what will 90 men earn in 40 days? *Ans.* £865.

If £90. in 5 months gain £2..5. what will £100. gain in 12 months? *Ans.* £6.

NOTE.—You multiply the 90 by 5, producing 450, and the 100 by 12, producing 1200; you may then divide 1200 by 450 (cancelling a cipher on each side) which you will find to go 2⅔ times, by which you multiply the £2..5. Or,

You may divide the middle term 45 shillings, into the first term, when multiplied together, which you will find to go 10 times, by which divide the third term, which you may do by cutting off one cipher from the 120,0 and taking the 120 left as shillings. *Ans.* £6.

If 25 *cwt.* of goods are carried 23 miles for £28..15. what should I pay for the carriage of 37½ *cwt.* if conveyed 40 miles? *Ans.* £75.

Obs. Operations of the above description may be wonderfully abbreviated by cancelling. Thus:

Take one of the foregoing questions.

If 5 men in 6 days earn £7..4..2. what will 90 men earn in 40 days?

Place the question thus:

$$\left.\begin{array}{l}\text{5 men.}\\ \text{6 days}\end{array}\right> \begin{array}{ccc}\pounds. & s. & d.\\ 7 & 4 & 2\end{array} \left<\begin{array}{l}\text{90 men divided by } 6 = 15\\ \text{40 days divided by } 5 = 8\end{array}\right.$$

$$\overline{120}$$

Here I see 6 into 90 will go 15 times; therefore I do away with the 6 and 90, and put 15, and 5 into 40 will go 8 times; I cancel both the one and the other. I then multiply the 15 and 8 together, which produce 120, and by ascertaining as many times £7..4..2. I have the Answer.

If 20 *cwt.* is conveyed 30 miles for £7..10..6. what must I pay for having 60 *cwt.* taken 5 miles? *Ans.* £3..15..3.

If 300 gallons of rum cost £181..5. what will 25 gallons cost? *Ans.* £15..2..1.

If 720 *cwt.* of steel cost £493..10..8. what will 18 *cwt.* cost? *Ans.* £12..6..9½.

A KEY

TO

FRYER'S GUIDE TO ARITHMETIC;

OR,

ANSWERS TO SUCH QUESTIONS AS ARE NOT CONTAINED IN THE WORK.

SIMPLE ADDITION.

Page	Quest.	Answers.
15	2	10138
16	3	12839
...	4	10465
...	5	102565
...	6	959993
...	7	9836268
...	8	Example.
...	9	862271
...	10	148114
...	11	14841069
...	12	14566
...	13	1949655
17	14	Example.
...	15	202417375
...	16	202417375
...	17	Example.
18	18	288138624
...	19	3093761337
...	20	2695843156
...	21	65646678
...	22	284898549
18	23	326865433
...	24	445486209
...	25	280818280
...	26	360095074
...	27	288069256
...	28	383048542
...	29	370351715
...	30	392717887
...	31	252385739
...	32	330137817
...	33	329416378
...	34	429500176
...	35	376970486
...	36	398575680
...	37	495003177
19	38	£872
...	39	8260 lbs.
...	40	18045 men.
...	41	26159 yards.
...	42	£37406
...	43	43922 days.

SIMPLE SUBTRACTION.

Page	Quest.	Answers.
20	1	Example.
...	2	Example.
21	3	Example.
21	4	Example.
22	5	5312
...	6	919

Page	Quest.	Answers.
22	7	4833
...	8	4137
...	9	4298
23	10	89 years.
...	11	5747 feet.
...	12	65248916562
...	13	123773079546
...	14	280157172480
...	15	237501637973
...	16	312972609394
...	17	621498404727
...	18	51868989490
...	19	28827373505
...	20	73911970364
...	21	145964299003
...	22	149864846558.
23	23	97028740868
...	24	211760457449
...	25	464393563978
...	26	197754090518
...	27	779536336800
...	28	794389000001
...	29	458679993792
...	30	103742865279
...	31	165191820893
...	32	551094646343
24	33	410898749178
...	34	1598749888
...	35	241692427847
...	36	379627900581
...	37	276187823097
...	38	334562106043

SIMPLE MULTIPLICATION.

Page	Quest.	Answers.
26	1 & 2	Examples.
27	3	9192
...	4	14124
...	5	22724
...	6	24915
...	7	7782
...	8	335757821
...	9	685146232
...	10	810347058
...	11	473654910
...	12	465366550
...	13	59565072
...	14	Example.
28	15	104223
...	16	164458
...	17	33322852
...	18	876169686
...	19	181564140
...	20	353267108
...	21	445999554
28	22	3968487216
...	23	25970447171
...	24	731421337920
...	25	9053803682280
...	26	Example.
...	27	26885596435656
...	28	59073063764
...	29	Example.
29	30 & 31	Examples.
30	32	42450603
...	33	46098276
...	34	7464480
...	35	6775296
...	36	7712594
...	37	Example.
...	38	663481364
...	39	1735292115
31	40 to 44	Examples, followed by Questions unworked.

SIMPLE DIVISION.

Page	Quest.	Answers.	Rem.	Page	Quest.	Answers.	Rem.
33	1	Example.		37	41	Example.	
...	2	7	0	39	42	738296	14
...	3	7	0	...	43	7280	22
...	4	8	0	...	44	63291	24
...	5	8	0	...	45	26578670	299
...	6	10	2	...	46	12621	187
...	7	6	6	...	47	8563	414
...	8	8	0	...	48	43488	2104
...	9	11	0	...	49	6270	62239
...	10	8	0	40	50	7745	169826
34	11	Example.		...	51	1344	7912
...	12	6	1	...	52, 53 & 54 Examples.		
...	13	7	3	41	55	Example.	
...	14	7	5	42	56	13236	15
...	15	5	6	...	57	6888	18
...	16	7	2	...	58	5076	0
...	17	9	2	...	59	43751	13
...	18	2	5	...	60	7834	22
...	19	4	1	...	61	6787	9
...	20	5	8	...	62	4482	4
...	21	7	3	...	63	31081	41
...	22	11	5	...	64	Example.	
35	23	Example.		43	65	201754174	0
36	24	1658860	2	...	66	397711742	2
...	25	14690814	2	...	67	176951055	19
...	26	11163054	0	...	68	163759831	14
...	27	811558236	2	...	69	194249313	2
...	28	407109243	6	...	70	1842638	43
...	29	76381440	1	...	71	11757789	45
...	30	10965873	2	...	72	2380952	16
...	31	80017485	10	...	73	12973322	46
...	32	81050203	0	...	74	5062122	6
...	33	67456374	3	...	75	6933728	92
...	34	361479149	4	...	76	252072	165
...	35	3040506007	10	...	77	166789	189
...	36	204010506	0	...	78	8429	449
...	37	20300904	12	...	79	1683	1686
...	38	16379593	7	...	80	118395387	3633
...	39	87326584	0	...	81	17074737	18935
...	40	4683634	4	...	82	1132978	294820

COMPOUND ADDITION.

Page	Quest.	Answers.		
45	1	Example.		
46	2 & 3	Examples.		
47	4	£154	15	4
...	5	200	16	11
...	6	197	12	10
...	7	181	0	2¾
...	8	304	2	0
...	9	1367	3	3
...	10	1470	8	10¼
...	11	1630	11	1½
...	12	3420	9	3
47	13	£8537	16	7¼
...	14	12871	17	2¾
...	15	11567	18	3
...	16	8223	11	6¼
...	17	16114	12	8½
48	18	14092	6	1¾
...	19	5503	16	5½
...	20	9095	18	5
...	21	15413	0	3½
...	22	19272	16	2¾

TROY WEIGHT.

Page	Quest.	Answers.
49	1	Example.
...	2	97 oz. 15 dwts. 1 gr.
...	3	164 oz. 11 dwts. 17 grs.
...	4	213 oz. 14 dwts. 23 grs.

AVOIRDUPOIS WEIGHT.

Page	Quest.	Answers.
49	1	Example.
50	2	150 cwt. 3 qrs. 2 lb.
...	3	157 cwt. 0 qrs. 27 lb.
...	4	198 cwt. 0 qrs. 5 lb.
...	5	199 tons 11 cwt. 3 qrs. 3 lb.

LONG MEASURE.

Page	Quest.	Answers.
50	1	4 per. 5 yds. 0 ft. 8 in.
...	2	217 m. 2 fur. 11 per. 2 yds. 1 ft. 4 in. 2 b. c.

DRY MEASURE.

Page	Quest.	Answers.
51	1	220 pks. 0 gals. 2 qts. 1 pt.
...	2	... ...

TIME.

Page	Quest.	Answer.
52	1	120 wks. 6 days 6 hrs. 58 min. 50 sec.

SUBTRACTION OF DIVERS DENOMINATIONS.

Page	Quest.	Answers.			
53	1 & 2	Examples.			
54	4	difference	£216	16	6
...	5		12	17	8½
...	6		24	13	9¾
...	7		48	3	2½
...	8		19	6	8¾
..	9	remains unpaid	38	14	1¼
55	10	balance of account	276	10	1

TROY WEIGHT.

Page	Quest.	Answers.
55	1	21 lbs. 5 oz. 17 dwts. 20 grs.
...	2	7 oz. 17 dwts. 23 grs.
...	3	1 oz. 14 dwts. 21 grs.
...	4	10 oz. 15 dwts. 23 grs.

AVOIRDUPOIS WEIGHT.

Page	Quest.	Answers.
55	1	10 cwt. 1 qr. 3 lb.
...	2	69 cwt. 2 qrs. 25 lb.
...	3	3 tons 17 cwt. 3 qrs. 13 lb. 1 oz. 11 dr.

LONG MEASURE.

Page	Quest.	Answer.
56	1	6 yds. 2 ft. 11 in.

LIQUID MEASURE.

Page	Quest.	Answers.
56	1	8 tuns 2 hhd. 54 gals.
...	2	1 hhd. 61 gals. 2 pts.

LAND MEASURE.

Page	Quest.	Answers.
56	1	8 acres 3 roods 5 perches.
...	2	31 acres 3 roods 16 perches.

COMMON TIME.

Page	Quest.	Answer.
56	1	3 yrs. 11 mo. 2 d. 18 hrs. 48 m. 11 sec.

MULTIPLICATION OF DIVERS DENOMINATIONS.

Page	Quest.		Answers.		
57	1 & 2		Examples.		
58	3		£7	9	7
...	4		5	13	2¼
...	5		3	15	10
...	6		7	11	8
...	7		22	3	1½
...	8		7	19	3
...	9		23	10	0
...	10		60	0	0
...	11		7	18	6½
...	12		103	4	4
...	13		47	15	6
...	14		21	19	0
			lbs.	*oz.*	*dwt.*
...	15	by 3	8	5	5
...	...	by 5	14	0	15
...	...	by 8	22	6	0
...	...	by 10	28	1	10
			yds.	*qrs.*	*na.*
...	16	by 4	19	2	0
...	...	by 6	29	1	0
...	...	by 9	43	3	2
			a.	*r.*	*p.*
...	17	by 4	11	1	24
...	...	by 6	17	0	16
...	...	by 9	25	2	24
59	18 & 19	Examples, explained in two or three ways.			

Page	Quest.		Answers.			
60	20	by 34	£62	10	11	
...	...	by 43	79	2	0½	
...	...	by 58	106	13	11	
...	...	by 76	139	16	2	
...	...	by 87	160	0	10½	
...	...	by 98	180	5	7	
...	...	by 104	191	6	4	
			t.	*cwt.*	*qrs.*	*lb.*
...	21	by 34	94	18	0	26
...	...	by 43	120	0	2	23
...	...	by 58	161	18	0	18
...	...	by 76	212	3	0	12
...	...	by 87	242	17	0	27
...	...	by 98	273	11	1	14
...	...	by 104	290	6	1	12
			lb.	*oz.*	*dwt.*	*grs.*
...	22	by 34	135	1	18	20
...	...	by 43	170	11	5	14
...	...	by 58	230	6	16	20
...	...	by 76	302	1	10	8
...	...	by 87	345	10	5	6
...	...	by 98	389	7	0	4
...	...	by 104	413	5	4	16
			a.	*r.*	*p.*	
...	23	by 34	124	0	16	
...	...	by 43	156	3	32	
...	...	by 58	211	2	32	
...	...	by 76	277	1	24	
...	...	by 87	317	2	8	
...	...	by 98	357	2	32	
...	...	by 104	379	2	16	
			yds.	*qrs.*	*na.*	
...	24	by 34	99	3	2	
...	...	by 43	126	1	1	
...	...	by 58	170	1	2	
..	...	by 76	223	1	0	
...	...	by 87	255	2	1	
...	...	by 98	287	3	2	
...	...	by 104	305	2	0	
61	25		Example.			
...	26		£1804	9	9	
...	27		9831	19	5¾	

Questions to exercise in MULTIPLICATION.

Page	Quest.	Answers.		
61	1	£0	13	8
...	2	4	12	6
...	3	2	3	3
...	4	1	18	4
...	5	1	12	3
...	6	0	7	2
...	7	0	18	2½
...	8	19	15	3¾
...	9	2	7	4¾
...	10	18	8	4
...	11	3	11	2
...	12	26	13	9
62	13	1635	7	6
		oz.	*dwt.*	*grs.*
...	14	9	13	12
...	15	£151	4	0
62	16	£23	2	0
...	17	8	9	0
...	18	30 *oz.* 4 *dwt.*		
...	19	£1	7	6
		oz.	*dwt.*	*grs.*
...	20	13	16	16
...	21	£14	17	1
...	22	31	4	8½
...	23	4	14	4¾
...	24	9	6	4
...	25	76	3	5
...	26	68	4	9
...	27	4	15	0¾
...	28	9	4	5¼
...	29	Answer given.		

DIVISION OF DIVERS DENOMINATIONS.

Page	Quest.		Answers.		
63	1		Example.		
...	2		£1	6	9
...	3		0	12	7
...	4		0	19	6
...	5		10	19	6
...	6		39	8	4
...	7		0	5	7¼
...	8		3	9	9¾
...	9		3	12	8
...	10		1 cwt. 0 qrs. 10 lb.		
...	11		1 yd. 2 qrs. 1 na.		
...	12		£15	4	8
64 & 65	13, 14 & 15		Examples.		
65	16	by 24	£2	0	7¾
...	...	by 25	1	19	0¼
...	...	by 48	1	3	$9\frac{15}{16}$
...	...	by 56	0	17	5 and 14 far. rem.
...	...	by 63	0	15	5 and 210 far. rem.
...	...	by 72	0	13	6 and 174 far. rem.
...	...	by 84	0	11	7 and 126 far. rem.
...	...	by 132	0	4	8 and 366 far. rem.

Page	Quest.		Answers.				
			cwt.	*qrs.*	*lb.*		
65	17	by 24	2	0	3	and 5 lb. over.	
...	...	by 25	1	3	22	and 3 lb. over.	
...	...	by 48	1	0	1	and 29 lb. over.	
...	...	by 56	0	3	13	and 21 lb. over.	
...	...	by 63	0	3	2	and 35 lb. over.	
...	...	by 72	0	2	19	and 53 lb. over.	
...	...	by 84	0	2	8	and 77 lb. over.	
...	...	by 132	0	1	13	and 41 lb. over.	
			oz.	*dwt.*	*grs.*		*grs.*
...	18	by 24	2	3	17	rem.	21
...	...	by 25	2	1	23	rem.	22
...	...	by 48	1	1	20	rem.	43
...	...	by 56	0	18	14	rem.	5
...	..	by 63	0	16	15	rem.	60
...	...	by 72	0	14	13	rem.	69
...	...	by 84	0	12	11	rem.	81
...	...	by 132	0	7	22	rem.	117
			a.	*r.*	*p.*		*perches.*
...	19	by 24	5	0	10	rem.	19
...	...	by 25	4	3	18	rem.	9
...	...	by 48	2	2	5	rem.	19
...	...	by 56	2	0	27	rem.	27
...	...	by 63	1	3	28	rem.	55
...	...	by 72	1	2	30	rem.	19
...	...	by 84	1	1	31	rem.	55
...	...	by 132	0	3	17	rem.	55
66	20 & 21	Examples.					*far.*
67	22	by 29	£84	17	8½	rem.	3
...	...	by 39	63	2	4¾	rem.	9
...	...	by 47	52	7	6¼	rem.	7
...	...	by 58	42	8	10¼	rem.	4
...	...	by 67	36	14	9¾	rem.	57
...	...	by 79	31	3	2½	rem.	8
...	...	by 143	17	4	3¼	rem.	139
...	...	by 476	5	3	5	rem.	350
...	...	by 740	3	6	6¼	rem.	394
...	...	by 846	2	18	2¼	rem.	336
...	...	by 947	2	11	11¾	rem.	449

Questions showing the use of DIVISION.

Page	Quest.	Answers.
69	1 to 7	Answers given.
...	8	£5 11 11¼ rem. 50 farthings.
70	9	10 oz. 15 dwt. 14 grs. rem. 88 grs.
71	10	17 cwt. 2 qrs. 11 lb. rem. 25 lbs.
...	11	36 ells 1 qr. 3 nails. rem. 1 nail.
...	12	2 yards 1 foot 3½ inches.
...	13	6s. 4½d. per ounce.
...	14	12s. 5d. per yard.
...	15	2s. 3½d. per bottle.
...	16	Answer given, see "Directions to Learner."
...	17	15s. 2d. per yard.
...	18	3s. 5¼d. per yard.
...	19	10¼d. per lb.
...	20	8¼d. per lb.
...	21	1¼d. each orange.
...	22	10s. 7¼d. per lb.
...	23	16 dwt. 11 grains a spoon.
...	24	23½ lb. the loaf.

REDUCTION.

Page	Quest.	Answers.
73	1, 2, 3, 4 & 5	Examples.
74	6	380 shillings, 4560 pence, and 18240 farth.
...	7	1155 shillings, 13860 pence, and 55440 far.
...	8	52405 farthings.
...	9	37245 halfpence.
...	10	22736 pence.
...	11	5726
...	12	2767 threepences, and 8301 pence.
...	13	2879 groats, 11516 pence, and 46064 farth.
...	14	4343 sixpences.
...	15	216 crowns, 432 half-crowns, 1080 shillings, 2160 sixpences, and 12960 pence.
75	16	1493¾ shillings, 5975 threepences, and 71700 farthings.
...	17	2880 pence, 240 shillings, and 12 pounds.
...	18	£74. 10s.
...	19	100 guineas.
...	20	210 pounds.

Page	Quest.	Answers.
75	21	2410 crowns, and £602. 10s.
...	22	77760 groats, and 25920 shillings; 5184 crowns, and 1296 pounds.
...	23	282 pieces.
...	24	101169 groats, 33723 shillings, 13489 half-crowns, and 6 pence over, 6744 crowns, and 3 shillings over, 1686 pounds, and 3 shillings over.
76	25	£602., and 11 shillings over.
...	26	5292 pence.
...	27	22¾, or 22 guineas, and 189 pence over.
...	28	1685 threepences, and 1 penny over.
...	29	210 dollars, and 45 pence over.
...	30	337 pieces, and 4*s*. 4*d*. over.
...	31	316 times.
...	32	10*s*. 5*d*. each, and 75 pence over.

TROY WEIGHT.

Page	Quest.	Answers.
76	33	Example.
...	34	2039040 grains.
...	35	1735 pennyweights.
77	36	2779992 grains.
...	37	354 pounds.
...	38	7 lbs. 2 oz. 15 dwts.
...	39	482 lbs. 7 oz. 13 dwts.

AVOIRDUPOIS WEIGHT.

Page	Quest.	Answers.
77	40	Example.
...	41 to 44	Answers given.

CLOTH MEASURE.

Page	Quest.	Answers.
77	45	1223 nails.
...	46	75 yards, and 7 nails over.
78	47	576 ells English.
...	48	720 yards.
...	49	960 ells Flemish.
...	50	576 ells English.

LAND MEASURE.

Page	Quest.	Answers.
78	51	Example.
...	52 & 53	Answers given.
...	54	21 roods, and 5 perches over.
...	55	2278 perches.

LIQUID MEASURE.

Page	Quest.	Answers.
79	56	Examples.
...	57	510 gallons.
...	58	466 gallons, 1864 quarts, and 3728 pints.
...	59	53 hhds. 58 gals. 3 pints, or 27179 pints.

DRY MEASURE.

Page	Quest.	Answers.
79	60	Example.
...	61	80 pecks, and 1 gallon over.
...	62	726½ bushels.
...	63	2936 bushels.
...	64	14167 pecks.
...	66	121 quarters, and 26 pecks over.
...	67	99868 pecks.
...	68	64 quarts.

TIME.

Page	Quest.	Answers.
80	69	Example.
...	70	216 hours, or 12960 minutes.
...	71	148785 minutes.
...	72	755 days.
...	73	17 days, 2 hours, 14 minutes, 10 seconds.

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